

Supplementary material to

Modeling the biodiesel production using the wheat straw ash as a catalyst

Ana Veličković¹, Jelena Avramović¹, Milan Kostić², Jugoslav B. Krstić³, Olivera Stamenković² i Vlada B. Veljković^{2,4},

¹University in Priština - Kosovska Mitrovica, Faculty of Technical Sciences, Knjaza Miloša 7, 38220, Serbia

²Faculty of Technology, University of Niš, Bulevar oslobođenja, 16000 Leskovac, Serbia

³University of Belgrade – Institute of Chemistry, Technology and Metallurgy, Belgrade, Serbia

⁴The Serbian Academy of Sciences and Arts, Knez Mihailova 35, 11000 Belgrade, Serbia

Table S-1. Experimental matrix (Replicates: runs 13/14, 27/28, 41/42, 55/56, and 69/7.)

| Run | Factor A | Factor B | Factor C | Factor D | Content of FAME, % (Y) |
|-----|----------|----------|----------|----------|------------------------|
| 1 | 55 | 15 | 18 | 60 | 23.7 |
| 2 | 65 | 15 | 18 | 60 | 67.4 |
| 3 | 55 | 15 | 24 | 60 | 37.2 |
| 4 | 65 | 15 | 24 | 60 | 36.5 |
| 5 | 55 | 10 | 21 | 60 | 49.4 |
| 6 | 65 | 10 | 21 | 60 | 41.6 |
| 7 | 55 | 20 | 21 | 60 | 15.6 |
| 8 | 65 | 20 | 21 | 60 | 11.2 |
| 9 | 60 | 10 | 18 | 60 | 73.8 |
| 10 | 60 | 10 | 24 | 60 | 40.9 |
| 11 | 60 | 20 | 18 | 60 | 32.2 |
| 12 | 60 | 20 | 24 | 60 | 18.2 |
| 13 | 60 | 15 | 21 | 60 | 74.0 |
| 14 | 60 | 15 | 21 | 60 | 81.9 |
| 15 | 55 | 15 | 18 | 90 | 45.3 |
| 16 | 65 | 15 | 18 | 90 | 84.8 |
| 17 | 55 | 15 | 24 | 90 | 68.1 |
| 18 | 65 | 15 | 24 | 90 | 85.9 |
| 19 | 55 | 10 | 21 | 90 | 73.5 |
| 20 | 65 | 10 | 21 | 90 | 68.4 |
| 21 | 55 | 20 | 21 | 90 | 35.7 |
| 22 | 65 | 20 | 21 | 90 | 25.9 |
| 23 | 60 | 10 | 18 | 90 | 91.7 |
| 24 | 60 | 10 | 24 | 90 | 64.2 |
| 25 | 60 | 20 | 18 | 90 | 59.7 |
| 26 | 60 | 20 | 24 | 90 | 35.2 |
| 27 | 60 | 15 | 21 | 90 | 91.1 |
| 28 | 60 | 15 | 21 | 90 | 93.1 |
| 29 | 55 | 15 | 18 | 120 | 57.8 |
| 30 | 65 | 15 | 18 | 120 | 92.4 |
| 31 | 55 | 15 | 24 | 120 | 76.0 |
| 32 | 65 | 15 | 24 | 120 | 94.9 |
| 33 | 55 | 10 | 21 | 120 | 85.3 |
| 34 | 65 | 10 | 21 | 120 | 76.9 |
| 35 | 55 | 20 | 21 | 120 | 51.9 |
| 36 | 65 | 20 | 21 | 120 | 23.0 |
| 37 | 60 | 10 | 18 | 120 | 95.8 |
| 38 | 60 | 10 | 24 | 120 | 82.3 |
| 39 | 60 | 20 | 18 | 120 | 74.8 |
| 40 | 60 | 20 | 24 | 120 | 52.5 |
| 41 | 60 | 15 | 21 | 120 | 95.0 |
| 42 | 60 | 15 | 21 | 120 | 95.7 |
| 43 | 55 | 15 | 18 | 180 | 79.9 |
| 44 | 65 | 15 | 18 | 180 | 97.5 |
| 45 | 55 | 15 | 24 | 180 | 93.4 |



| Run | Factor A | Factor B | Factor C | Factor D | Content of FAME, % (Y) |
|-----|----------|----------|----------|----------|------------------------|
| 46 | 65 | 15 | 24 | 180 | 98.4 |
| 47 | 55 | 10 | 21 | 180 | 95.4 |
| 48 | 65 | 10 | 21 | 180 | 94.1 |
| 49 | 55 | 20 | 21 | 180 | 79.3 |
| 50 | 65 | 20 | 21 | 180 | 63.9 |
| 51 | 60 | 10 | 18 | 180 | 98.1 |
| 52 | 60 | 10 | 24 | 180 | 91.4 |
| 53 | 60 | 20 | 18 | 180 | 91.3 |
| 54 | 60 | 20 | 24 | 180 | 78.6 |
| 55 | 60 | 15 | 21 | 180 | 97.5 |
| 56 | 60 | 15 | 21 | 180 | 96.5 |
| 57 | 55 | 15 | 18 | 240 | 93.1 |
| 58 | 65 | 15 | 18 | 240 | 97.5 |
| 59 | 55 | 15 | 24 | 240 | 98.2 |
| 60 | 65 | 15 | 24 | 240 | 98.4 |
| 61 | 55 | 10 | 21 | 240 | 97.8 |
| 62 | 65 | 10 | 21 | 240 | 94.1 |
| 63 | 55 | 20 | 21 | 240 | 90.6 |
| 64 | 65 | 20 | 21 | 240 | 78.5 |
| 65 | 60 | 10 | 18 | 240 | 98.6 |
| 66 | 60 | 10 | 24 | 240 | 94.9 |
| 67 | 60 | 20 | 18 | 240 | 96.1 |
| 68 | 60 | 20 | 24 | 240 | 91.2 |
| 69 | 60 | 15 | 21 | 240 | 98.4 |
| 70 | 60 | 15 | 21 | 240 | 96.7 |

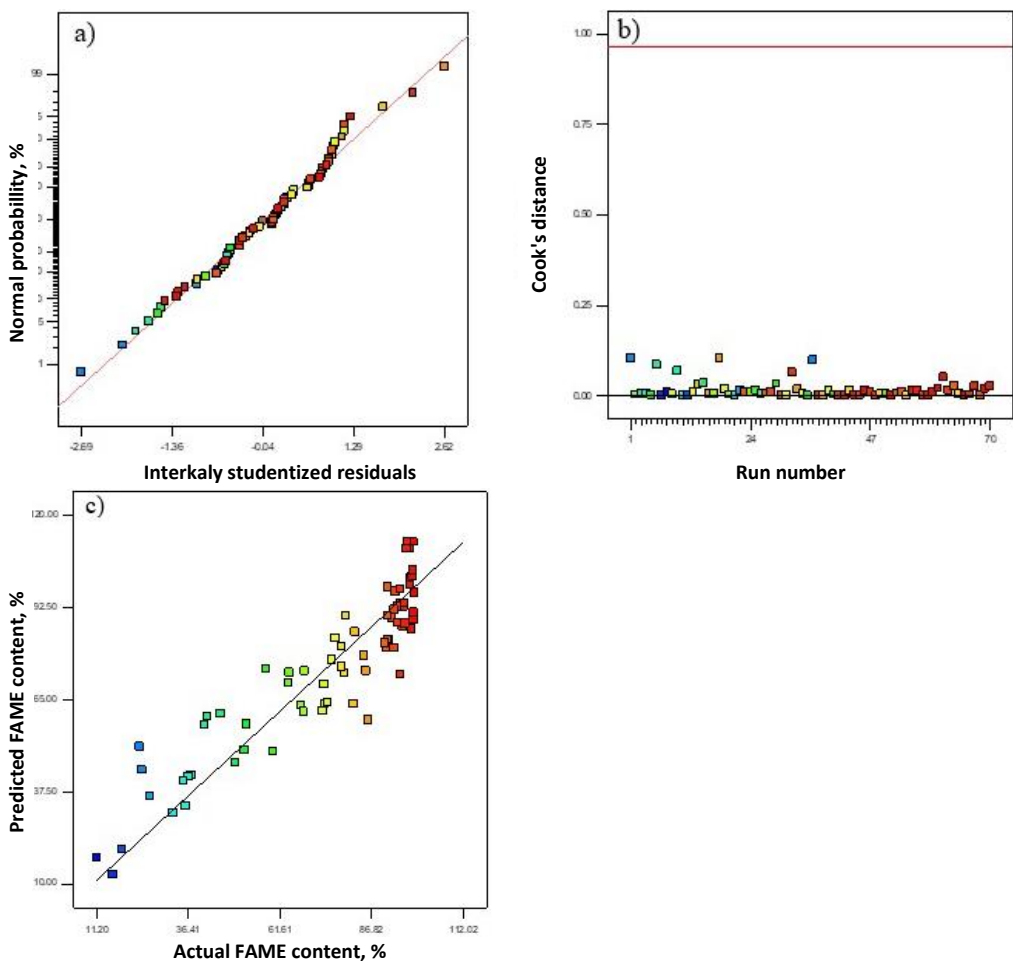


Fig. S-1. Normal probability plot of residuals (a), Cook's distance (b), and predicted versus actual values of FAME content for the developed quadratic model



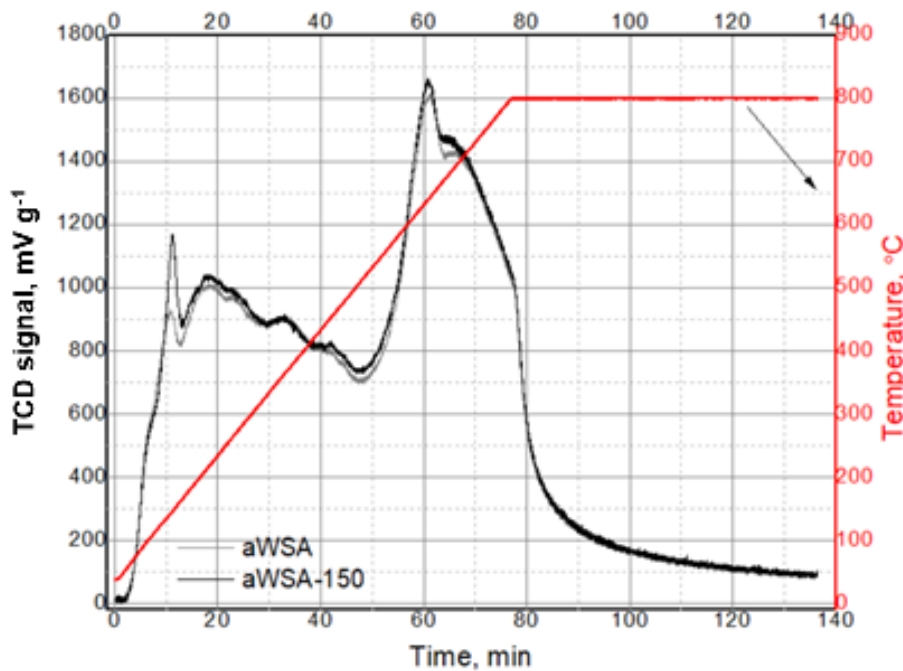


Fig. S-2 Checking the reproducibility of TCD for the differently aged fWSA sample; aWSA aging time 40 days and aWSA-150 aging time 150 days

Adequacy of the historical data design-based models

Table S-2. Results of sequential model sum of squares test

| Source | Sum of squares | df | Mean square | F-value | p-value | Remark |
|---------------------|----------------|----|-------------|---------|----------|------------------|
| Mean vs: Total | 380351.0 | 1 | 380351.0 | | | |
| Linear vs. Mean | 27342.1 | 4 | 6835.5 | 24.7 | < 0.0001 | |
| 2FI vs. Linear | 2104.2 | 6 | 350.7 | 1.3 | 0.271 | |
| Quadratic vs 2FI | 8765.7 | 4 | 2191.4 | 16.9 | < 0.0001 | Suggested |
| Cubic vs. Quadratic | 5597.1 | 13 | 430.5 | 11.6 | < 0.0001 | Aliased |
| Residual | 1553.7 | 42 | 37.0 | | | |
| Total | 425713.7 | 70 | 6081.6 | | | |

The test selects the highest order polynomial, where the additional terms are significant, and the model is not aliased.

Table S-3. Results of lack of fit test

| Source | Sum of squares | df | Mean square | F value | p value | Remark |
|------------|----------------|----|-------------|---------|---------|------------------|
| Linear | 17985.3 | 60 | 299.8 | 42.3 | 0.0003 | |
| 2FI | 15881.1 | 54 | 294.1 | 41.5 | 0.0003 | |
| Quadratic | 7115.4 | 50 | 142.3 | 20.1 | 0.0016 | Suggested |
| Cubic | 1518.3 | 37 | 41.0 | 5.8 | 0.0288 | Aliased |
| Pure Error | 35.4 | 5 | 7.1 | | | |

It wants the selected model to have insignificant lack-of-fit.

Table S-4. Results of model summary statistics test. This test focuses on the model maximizing the R^2_{adj} . PRESS - Predicted Residual Sum of Squares for the model

| Source | Standard deviation | R^2 | R^2_{adj} | R^2_{pred} | PRESS | Remark |
|-----------|--------------------|-------|-------------|--------------|---------|------------------|
| Linear | 16.7 | 0.603 | 0.578 | 0.546 | 20592.8 | |
| 2FI | 16.4 | 0.649 | 0.590 | 0.528 | 21389.1 | |
| Quadratic | 11.4 | 0.842 | 0.802 | 0.754 | 11161.3 | Suggested |
| Cubic | 6.1 | 0.966 | 0.944 | 0.876 | 5643.3 | Aliased |



ANOVA results for the quadratic model

Final quadratic equation in terms of coded and actual factors

Coded factors:

$$Y = 103.68 + 1.70A - 11.60B - 2.43C + 24.21D - 2.22AB - 4.93AC - 3.03AD + 0.30BC + 7.31BD + 3.32CD - 13.07A^2 - 16.32B^2 - 2.60C^2 - 15.86D^2 \quad (S\ 1)$$

Actual factors:

$$Y = 2578.59 + 72.32A + 19.73B + 28.90C + 0.759D - 0.089AB - 0.329AC - 0.007AD + 0.020BC + 0.016BD + 0.012CD - 0.523A^2 - 0.653B^2 - 0.289C^2 - 0.002D^2 \quad (S\ 2)$$

Statistical criteria:

$$R^2 = 0.842, R^2_{\text{adj}} = 0.802, R^2_{\text{pred}} = 0.745, \text{MRPD} = \pm 14.6\%, \text{C.V.} = 15.5\%, \text{and Adeq. Prec.} = 18.8.$$

Table S-5. Optimal process conditions based on the quadratic equation

| A | B | C | D | Y / % |
|-------------|-------------|-------------|------------|-------------------------|
| 60.3 | 11.6 | 18.3 | 124 | 98.6^a |
| 61.1 | 12.3 | 18.4 | 118 | 98.6 |
| 60.5 | 16.6 | 18.5 | 149 | 98.6 |
| 63.2 | 15.2 | 18.5 | 138 | 98.6 |
| 60.1 | 11.3 | 18.6 | 127 | 98.6 |
| 56.4 | 14.3 | 18.8 | 167 | 98.6 |
| 57.3 | 10.3 | 20.2 | 220 | 98.6 |
| 64.0 | 12.3 | 21.1 | 149 | 98.6 |
| 64.5 | 13.3 | 22.0 | 170 | 98.6 |
| 62.4 | 10.8 | 22.2 | 151 | 98.6 |
| 64.1 | 14.6 | 22.8 | 183 | 98.6 |
| 58.6 | 13.8 | 22.8 | 138 | 98.6 |
| 62.3 | 15.1 | 22.9 | 155 | 98.6 |
| 63.1 | 16.2 | 23.0 | 185 | 98.6 |
| 57.1 | 13.4 | 23.1 | 149 | 98.6 |
| 62.9 | 13.9 | 23.6 | 162 | 98.6 |
| 55.4 | 16.1 | 23.6 | 188 | 98.6 |
| 63.2 | 11.4 | 23.9 | 219 | 98.6 |
| 56.9 | 11.6 | 24.0 | 165 | 98.6 |

^aThe selected optimal process conditions: temperature of 60.3 °C, catalyst loading of 11.6 % (based on the oil mass), methanol-to-oil molar ratio of 18.3, and reaction time of 124 min. Desirability = 1 in all runs.