Control of the size and compositional distributions in a milling process by using a reverse breakage matrix approach

Nemanja Bojanić[1], Aleksandar Fišteš[1], Tatjana Došenović[1], Aleksandar Takači[1], Mirjana Brdar[1], Kiyoshi Yoneda[2] and Dušan Rakić[1]

[1]Faculty of Technology, University of Novi Sad, Boulevard cara Lazara 1, 21000 Novi Sad, Serbia
[2]Faculty of Economics, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka 814-0180, Japan

Abstract

A method based on the reverse breakage matrix approach is proposed for controlling the effects that milling has on the particle size distribution and composition of the comminuted material. Applicability, possibilities, and limitations of the proposed method are tested on examples related to the process of wheat flour milling. It has been shown that the reverse matrix approach can be successfully used for defining the particle size distribution of the input material leading to the desired, predetermined particle size and compositional distribution in the output material. Moreover, we have illustrated that it is possible to simultaneously control both, input and output particle size distribution, together with the composition of the output material.

Keywords: Reverse problem; distribution of chemical compounds; particle size distribution; wheat flour milling.

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1. INTRODUCTION

In many industries, size reduction operation represents one of the crucial steps in the process, since material often occurs in an inadequate size for further processing, which needs to be reduced to a certain level [1,2]. The goal of comminution process depends on the further use of the ground material. The objective can be size reduction, but also the purpose can be more complex, if the aim of the process is to obtain particles from a heterogeneous raw material with specific shapes, composition, etc.

Milling of wheat is probably the prime example of such a process, since, besides the kernel size reduction, the goal is to achieve efficient segregation of the main anatomic parts of wheat kernel [3]. To accomplish this, the miller needs to exploit differences in mechanical properties between endosperm, bran and germ [4]. The most common way to attain this goal is to modify operating conditions such as the roll gap, roll speed, differential, feed rate, etc. In addition to process parameters, physico-chemical and structural characteristics, such as the wheat kernel size and hardness, significantly influence the comminution process [5]. Chemical analysis of output fractions can provide information on the effectiveness of the process, since the contents of proteins, ash, etc., vary in different wheat kernel anatomic parts [6]. For this purpose, the ash content in flour and intermediate products is considered as one of the main indicators that show how efficiently bran and germ are separated from the endosperm during the process [7,8].

Considering the importance of particle size distribution (PSD) in the output material from any milling operation, many efforts have been made to define a suitable model that would describe the process of particle breakage during comminution. Among various modeling approaches, such as the discrete element method and finite element method, population balance type of modeling (PBM) is probably most often used to mathematically describe the comminution process [9–13]. However, the time-continuous form of the PBMs requires continual intra-process sampling for monitoring the evolution of PSD over time, and this is often difficult and sometimes even impossible to achieve. Also, PBM has been restricted to particle size determination and not the compositional distribution in the milling output.

On the other hand, the time-discretized form of the PBM – breakage matrix approach, treats the entire process as a single breakage event, giving an overall relationship between the input and output PSDs [14]. Broadbent and Callcott [15–17] were first to develop a breakage matrix approach in order to relate the input and output particle size distribution during a comminution process. They described the relationship between the input size vector (*f*) and the resulting output size vector (*o*) by the matrix equation:

$$B_{m \times n} f_{n \times 1} = o_{m \times 1}. \tag{1}$$

The fact that vectors (*f*) and (*o*) represent the weight fractions of the input and output material, makes this approach applicable to practical experiments, and convenient for studies in which PSDs are measured using sieve analysis [5]. Elements of the breakage matrix can be determined based on narrow range size fractions of the inlet material and the obtained PSDs from the sieve analysis, thus forming the corresponding columns in the breakage matrix.

Once determined for one set of milling conditions, the breakage matrix cannot be used for another set of conditions [5]. This could be considered as a limitation of the breakage matrix approach. Also, the approach presumes that particle breakage is not influenced by any inter-particle interaction, which means that particles of the same size, milled on the same set of conditions are assumed to break in the same manner regardless if they belong to a mono-dispersed sample or a poly-dispersed mixture [18]. However, recent works investigated the application of modified breakage matrix methodology in order to characterize multi-particle interactions during breakage [12,14,19,20]. Thus, the breakage matrix approach presented by Eq. 1 as the empirical model is suitable for modeling comminution processes in the once-through type of mills, where the retention time of the milled particles is short. A prime example of this kind of mill is a roller mill for wheat flour milling.

Broadbent and Callcott [15–17] used the same sieve size for feed and product size distribution, which together with the assumption that all particles are broken during a single breakage event resulted in square breakage matrices (dimension *n×n*) where above diagonal values were equal to zero [19]. In practice, PSDs of the input and output material mostly occur in different size ranges. An improvement of the model was introduced by Campbell and Webb [18], which used different sieve sizes and different numbers of sieves for the inlet and outlet size distributions, thus obtaining non-square matrices, which were more accurate and more general. Applicability of this method was confirmed for predicting PSD of the stock following the first-break roller milling of wheat [5,18].

As mentioned before, for complete understanding of the milling process, sometimes it is important to obtain data on PSD, together with the composition of the broken particles. Since the columns of the breakage matrix are determined by sieve analysis, simultaneously the samples are obtained in which the content of certain chemical compound can be determined. Fistes and Tanovic [21] in a case study of first break roller milling of wheat, defined and confirmed the form of a breakage matrix for predicting compositional distribution in output fractions, along with their size distribution:

$$\begin{bmatrix} Y_{11}/o_1 & Y_{12}/o_1 & \cdots & Y_{1n}/o_1 \\ Y_{21}/o_2 & Y_{22}/o_2 & \cdots & Y_{2n}/o_2 \\ \vdots & \vdots & \vdots & \vdots \\ Y_{m1}/o_m & Y_{m2}/o_m & \cdots & Y_{mn}/o_m \end{bmatrix} \cdot \begin{bmatrix} f_1 P_1 \\ f_2 P_2 \\ \vdots \\ f_n P_n \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix}, \tag{2}$$

This matrix can be used for prediction of the contents of some chemical compounds in the outlet fractions of the comminuted material. The values of Y_{ij} which represent the weight fractions of a certain compound in the corresponding output size fraction, are the fixed elements of the breakage matrix. Content of a defined chemical compound in the milling output size fractions (p_i) also depend on the input size distribution (f_j) milling output particle size distribution (o_i) and the content of the observed compound in the different input size fractions (P_j).

Based on the compositional breakage matrix, Galindez – Najera *et al.* [22] defined a continuous breakage equation for calculating composition of particles, alongside with prediction of PSD. However, they observed the compositional distribution in terms of distribution of botanical parts of wheat kernel rather than a chemical distribution in the fractions of the milling output.

A reverse problem in the breakage matrix context considers the possibility of calculating the input particle size distribution that would lead to desired output PSD. Especially, attention must be given to positivity of solutions, since, f_j, $1 \leq j \leq n$, are the mass fractions. This problem is solved by a precision approach using linear programming while approximative solutions are obtained by numerical methods [23-25]. To achieve the best-fit solution, the LL'

decomposition [26], singular value decomposition (SVD) [23] or QR decomposition ([24]) could be used. Also, this problem could be solved by using standard methods (*e.g.* the least squares method) in which negative solutions are set to 0. This solution usually cannot be considered as an optimal one. In order to avoid negative solutions, another option is to use some estimates of error in which the positivity condition is integrated in the error approximation norm. A norm defined by the least semilogs [27] is used for this purpose.

Once a breakage matrix is defined for a certain set of milling conditions it cannot be employed for another set of parameters [5]. Therefore, application of the breakage matrix approach for the control of a milling process implies manipulation of the particle size distribution of the milled material in order to obtain optimum results.

The main goal of this paper is to connect the afore mentioned issues and to predict the compositional distribution in the milling output by using the reverse breakage matrix approach. More precisely, the paper aims to investigate possibilities and limitations of reverse matrices in defining the PSD of the input material to a milling operation, which would provide the desired compositional distribution together with the desired PSD of the output material. Also, the aim is to show flexibility of the suggested method, which allows simultaneous PSD control of both input and output materials together with the compositional control of the output. Mathematical procedures presented in the following section "Theoretical part" are illustrated in the section "Results and discussion" by using four examples. The approach is considered as a general case and illustrated with the examples related to wheat flour milling as a paradigm of a technological process with a clearly expressed need for controlling the composition of the milling output fractions.

2. MATERIALS AND METHODS

Physicochemical and structural characteristics of the wheat sample that was used in the experiment were determined as: 14.8 % protein, 73.5 % vitreous, and 10.4 % moisture content at the bulk density of 843 kg/m^3. Protein content and moisture content were determined according to the standard methods of the International Association for Cereal Science and Technology (ICC standard methods [28,29]). Virtuousness of wheat was determined on the basis of observation of grain cross-section and classification into categories: fully vitreous, ¾ vitreous, ½ vitreous, ¼ vitreous and completely floury [30]. To obtain the cross-section of the grain farinatom was employed. The bulk density was determined using a Chopper scale and special tables attached to each scale [30].

The wheat sample was separated into fractions by using the sorting machine Sortimat (Perten AB, Sweden). The machine was equipped with two metal sieves with opening sizes of 2.8 ≅ 20 mm and 2.8·20 mm. By these sieves, the wheat sample was separated into three size fractions: 1 > 2.8 ≅ 20 mm, 2 - 2.8 ≅ 20 mm / 2.5 ≅ 20 mm and 3 < 2.5≅20 mm. For each size fraction, moisture and ash contents have been determined, according to the standard methods of the International Association for Cereal Science and Technology (ICC standard methods [29,31]). Values of the moisture content for size fractions were: 10.5, 10.2 and 10.4 %, respectively, while the ash contents were: 1.4, 1.47 and 1.58 %, respectively.

Conditioning treatment prior to milling was customized to correspond to structural characteristics of the wheat and it was conducted in three steps. In the first step, the samples (batches of 0.5 kg of the wheat kernel size fractions) were conditioned to 13.5 % moisture for 20 h. After that, the moisture content was raised to 16 % for 12 h. In the final step, 30 min before milling, the moisture content in the samples was raised for another 0.5 %. Conditioning is an important step in wheat preparation for milling and it is done in order to emphasize the differences in the structural and mechanical properties that exist between anatomical parts of the grain [3].

After conditioning, samples were milled by using the laboratory roll stand Variostuhl, model C Ex 2 (Miag, Braunschweig, Germany), with the milling parameters shown in Table 1.

Table 1. Operating parameters of the milling process

Roll diameter, mm	Roll length, mm	Flutes, cm^{-1}	Flute spiral, %	Flute angles, °
250	100	3.2	8	40/70
Roll disposition	Fast roll speed, m s^{-1}	Differential	Roll gap, mm	Feed rate, kg m^{-1} s^{-1}
dull/dull	6	2.5	0.5	0.21

The entire milled stock (five batches of 100 g) was sieved for 3 min on the Bühler laboratory sifter (gyratory in a horizontal plane), model MLU-300 (Uzwil, Switzerland), and separated into eight fractions by using square aperture sieves of sizes 2000, 1180, 850, 600, 450, 300 and 150 μm, along with a bottom collecting pan (fractions 1–8 respectively) and rubber balls as cleaners. In all of the eight size fractions moisture and ash contents were determined [29,31] and, along with the sieve analysis data, used for construction of breakage matrices for predicting particle size distribution B and ash content in the milling output size fractions *Y* (represented by matrices (18)). Particle size distribution was determined on the dry matter basis.

3. THEORETICAL PART

Since columns of the matrix *B*, as well as vectors *f* and *o,* defined in eq. (1), represent PSD, following conditions hold:

$$\sum_{j=1}^{n} f_j = 1, \ \sum_{i=1}^{m} o_i = 1, \text{ and } \sum_{i=1}^{m} b_{ij} = 1, \ 1 \leq j \leq n. \tag{3}$$

By applying eqs. (1) and (3) it is easy to obtain that

$$\min_{1 \leq j \leq n} b_{ij} \leq o_i \leq \max_{1 \leq j \leq n} b_{ij} \ 1 \leq i \leq m \tag{4}$$

The physical meaning of the condition (4) is that a mass fraction of any size fraction of the output (o_i) must be between the minimum and maximum in the corresponding row of the breakage matrix.

As it was mentioned previously in text, Fistes and Tanović [21] defined the matrix eq. (2) for predicting the compositional distribution in the output material from the milling operation where the following conditions hold:

$$Y_{ij} = \frac{b_{ij} \cdot p_{ij}}{p_j}, \ 1 \leq i \leq m, \ 1 \leq j \leq n, \ \sum_{i=1}^{m} Y_{ij} = 1, \ 1 \leq j \leq n \tag{5}$$

By substituting appropriate elements of matrix equations (1) and (2) the following matrix equation is obtained:

$$\begin{bmatrix} b_{11} \cdot p_1 - Y_{11} \cdot P_1 & b_{12} \cdot p_1 - Y_{12} \cdot P_2 & \cdots & b_{1n} \cdot p_1 - Y_{1n} \cdot P_n \\ b_{21} \cdot p_2 - Y_{21} \cdot P_1 & b_{22} \cdot p_2 - Y_{22} \cdot P_2 & \cdots & b_{2n} \cdot p_2 - Y_{2n} \cdot P_n \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} \cdot p_m - Y_{m1} \cdot P_1 & b_{m2} \cdot p_m - Y_{m2} \cdot P_2 & \cdots & b_{mn} \cdot p_m - Y_{mn} \cdot P_n \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{6}$$

where:

$$\min_{1 \leq j \leq n} \frac{Y_{ij} \cdot P_j}{b_{ij}} \leq p_i \leq \max_{1 \leq j \leq n} \frac{Y_{ij} \cdot P_j}{b_{ij}} \ 1 \leq i \leq m \tag{7}$$

The physical meaning of the condition (7) is that the concentration of some compound p_i in any of the output fractions must be between the minimum and maximum, which is determined by the elements in the corresponding row of the breakage matrix.

By eq. (6), for a given set of inputs (f_1, f_2, …, f_n), the concentration of any chemical compound in outputs can be calculated as:

$$p_i = \frac{\sum_{j=1}^{n} Y_{ij} P_j f_j}{b_{ij} f_j} \ 1 \leq i \leq m \tag{8}$$

The "reverse" problem, in this context, means calculation of the input PSD, which simultaneously leads to desired output PSD and concentration of some compound in output fractions. This is implied by the direct connection between p_i and o_i given by eq. (2). Hence, there are three directions in a study to find the input PSD for a chosen:

    (A) output PSD,
    (B) concentration of a chemical compound in output fractions, and
    (C) combination of the output PSD and concentration of the selected compound in output fractions.

Previous studies of the reverse problem in the breakage matrix context focused on finding the input PSD to a milling operation, which would result in the desired output PSD. In the case (A) the following approach is expanded with the possibility to simultaneously choose both output and input PSDs.

In the case (B), by using eq. (6), input fractions can be expressed as nonlinear function of the selected compound content in the output fractions, *i.e.*:

$$f_j = F_j(p_1, p_2, \ldots, p_{n-1}) , \quad j = 1, 2, \ldots, n-1 , \quad f_n = 1 - \sum_{j=1}^{n-1} f_j \qquad (9)$$

Then, acceptable values of compound contents are provided by a system of n nonlinear inequalities:

$$F_j(p_1, p_2, \ldots, p_{n-1}) \geq 0 , \quad j = 1, 2, \ldots, n-1 \ \text{ and } \ \sum_{j=1}^{n-1} F_j(p_1, p_2, \ldots, p_m) \leq 1 \qquad (10)$$

Clearly, $n - 1$ compound contents from the set $p_1, p_2, \ldots, p_m$ can be arbitrarily chosen.

In particular, functions defined in eq. (9), when the dimension of breakage matrices B and Y is $m \times 3$, are:

$$f_1(p_1, p_2) = \frac{A_{12} p_1 p_2 + A_1 p_1 + A_2 p_2 + A_0}{B_{12} p_1 p_2 + B_1 p_1 + B_2 p_2 + B_0}$$

$$f_2(p_1, p_2) = \frac{C_{12} p_1 p_2 + C_1 p_1 + C_2 p_2 + C_0}{B_{12} p_1 p_2 + B_1 p_1 + B_2 p_2 + B_0} \qquad (11)$$

$$f_3(p_1, p_2) = \frac{(B_{12} - A_{12} - C_{12}) p_1 p_2 + (B_1 - A_1 - C_1) p_1 + (B_2 - A_2 - C_2) p_2 + B_0 - A_0 - C_0}{B_{12} p_1 p_2 + B_1 p_1 + B_2 p_2 + B_0}$$

where the coefficients are precisely shown in the Appendix.

Now, the system (10) consists of three nonlinear inequalities:

$$f_1(p_1, p_2) \geq 0 , \ f_2(p_1, p_2) \geq 0 \text{ and } f_1(p_1, p_2) + f_2(p_1, p_2) \leq 1 \qquad (12)$$

with a 3-dimensional representation of acceptable values of the selected chemical compound content.

As it is suggested in the study by Fistes and coworkers [24] instead of choosing the values p_1, p_2 simultaneously, they can be chosen successively. Then, by the condition (7), choosing p_1 leads to an acceptable range for p_2, as a solution of the system (12) with 2-dimensional geometrical interpretation.

Alternatively, an acceptable range for p_2 can be found in the following way. By placing the chosen value for p_1 in the matrix equation (6) a linear dependence between f_1 and f_2 is obtained. Intersections of this line with the f_1-axis and the line $f_1 + f_2 = 1$ define the acceptable range (a,b) for f_1. Now, the function $f_1 = F_1(p_1, p_2)$ with the known p_1 is observed. If it is monotone in the interval (a,b), then the values a and b define the acceptable range for p_2.

In the case (C), by eqs. (1) and (6), input weight fractions (f) can be expressed by the output weight fractions (o) and the compound content in the output size fractions (p) *via* nonlinear functions:

$$f_j = F_j(o_1, o_2, \ldots, o_k, p_1, p_2, \ldots, p_s) , \quad j = 1, 2, \ldots, n-1 , \quad f_n = 1 - \sum_{j=1}^{n-1} f_j \qquad (13)$$

where $k + s = n - 1$. The solution (acceptable values of weight fractions and the compound content in the output) is provided by the system of n nonlinear inequalities:

$$F_j(o_1, o_2, \ldots, o_k, p_1, p_2, \ldots, p_s) \geq 0, \ j = 1, 2, \ldots, n-1, \ \sum_{j=1}^{n-1} F_j(o_1, o_2, \ldots, o_k, p_1, p_2, \ldots, p_s) \leq 1 \qquad (14)$$

Let the dimension of the breakage matrix be $m \times 3$. By eqs. (1) and (6) input fractions can be expressed as nonlinear functions of o_1 and p_1 as:

$$f_1(o_1, p_1) = A_{11} o_1 p_1 + A_1 o_1 + A_0 , \ f_2(o_1, p_1) = B_{11} o_1 p_1 + B_1 o_1 + B_0 \qquad (15)$$

Then, the acceptable values of o_1 and p_1 are represented in 3 dimensions, as a solution of the system with three nonlinear inequalities:

$$f_1(o_1, p_1) \geq 0 , \ f_2(o_1, p_1) \geq 0 \text{ and } f_1(o_1, p_1) + f_2(o_1, p_1) \leq 1 \qquad (16)$$

If, o_1 is chosen according to eq. (4), then eq. (12) becomes the system of three linear inequalities in 2D where the intersection of lines with p_1-axis defines the acceptable range for p_1.

Alternatively, o_1 can be chosen and by eq. (1) the linear relationship between input fractions is obtained:

$$f_2 = \frac{b_{11} - b_{13}}{b_{13} - b_{12}} f_1 + \frac{o_1 - b_{13}}{b_{12} - b_{13}} \qquad (17)$$

Now, by the first equation in (6):

$$p_1 = \frac{Y_{11}P_1(b_{12}-b_{13})+Y_{12}P_2(b_{13}-b_{11})+Y_{13}P_3(b_{11}-b_{12})}{o_1(b_{12}-b_{13})}\cdot f_1 + \frac{Y_{12}P_2(o_1-b_{13})+Y_{13}P_3(b_{12}-o_1)}{o_1(b_{12}-b_{13})} \tag{18}$$

Intersection of the line (17) with the f_1-axis and the line $f_1 + f_2 = 1$ defines the acceptable range (a,b) for f_1:

$$a = \max\left\{0, \frac{o_1-b_{12}}{b_{11}-b_{12}}\right\},\quad b = \min\left\{\frac{o_1-b_{13}}{b_{11}-b_{13}},1\right\}. \tag{19}$$

If a and b are placed instead of f_1 in eq. (18) the acceptable range for p_1 is obtained.

Also, by the condition (7), p_1 can be chosen so that f_1 and f_2 are expressed as functions of o_1:

$$f_1(o_1) = A_1 o_1 + A_0 ,\ f_2(o_1) = B_1 f_1(o_1) + B_0 ,\ f_3(o_1) = 1 - f_1(o_1) + f_2(o_1) \tag{20}$$

Now, conditions $f_1(o_1) \geq 0$, $f_2(o_1) \geq 0$ and $f_1(o_3) \geq 0$ define the acceptable range for o_1.

Similarly, lines

$$f_2 = B_1 f_1 + B_0 \ \text{and}\ f_2 = 1 - f_1 \tag{21}$$

determine the acceptable range for f_1, which defines the acceptable range for o_1 by the relation:

$$o_1(f_1) = \frac{1}{A_1}f_1 - \frac{A_0}{A_1} \tag{22}$$

On the other hand, all compound contents in output fractions (case B) or all output fractions and compound contents in all output fractions (case C) can be chosen and the best approximation can be found. Then, in the general case, the overdetermined system given in the matrix form:

$$C_{m\times n}\ X_{n\times 1} = D_{m\times 1} \tag{23}$$

is observed. The norm of least semilogs [25]:

$$\|D-CX\|_{\text{semilog}} = \sum_{i=1}^{m}\frac{D_i-C_iX}{C_iX}\log\frac{D_i}{C_iX} \tag{24}$$

where C_i denote the ith row of the matrix C, can be used in solving the positive inverse linear problem. Listed cases are studied for $m\times 3$ dimension of the breakage matrix.

In the case B, by eq. (2), the semilog norm

$$\sum_{i=1}^{m}\frac{B_i-A1_i f_1-A2_i f_2}{A1_i f_1 + A2_i f_2}\log\frac{B_i}{A1_i f_1 + A2_i f_2} \tag{25}$$

is defined, where

$$A1_i = b_{i1}p_i - Y_{i1}P_1 - b_{i3}p_i + Y_{i3}P_3$$
$$A2_i = b_{i2}p_i - Y_{i2}P_2 - b_{i3}p_i + Y_{i3}P_3 \tag{26}$$
$$B_i = -b_{i3}p_i + Y_{i3}P_3$$

In the case C, by using eq. (2) the proposed norm is:

$$\sum_{i=1}^{m}\frac{p_i o_i - YP_i f}{YP_i f}\log\frac{p_i o_i}{YP_i f} \tag{27}$$

where i^{th} row of the matrix

$$YP = \begin{bmatrix} Y_{11}\cdot P_1 & Y_{12}\cdot P_2 & Y_{13}\cdot P_3 \\ Y_{21}\cdot P_1 & Y_{22}\cdot P_2 & Y_{23}\cdot P_3 \\ \vdots & \vdots & \vdots \\ Y_{m1}\cdot P_1 & Y_{m2}\cdot P_2 & Y_{m3}\cdot P_3 \end{bmatrix} \tag{28}$$

is denoted by YP_i.

4. RESULTS AND DISCUSSION

The applicability of the proposed approach was tested by examples related to first break milling of wheat. As it was mentioned earlier in the text wheat flour milling is probably the prime example of a grinding process in which the composition of ground particles is as important as their size. In that sense, ash determination is probably the most widely used measurement of the wheat milling efficiency. Ash is concentrated in the bran and it is a relatively accurate index of the separation of endosperm from pericarp and germ in any particular stream or flour.

The sieve analysis data of the stock obtained by the first break milling of wheat kernel size fractions are presented in the form of a breakage matrix. It is a case with 3 input fractions and 8 output fractions from a milling operation ($n=3$, $m=8$). The output size distributions represent the columns of the matrix B. The obtained output size fractions were analyzed for the ash content and together with the sieve analysis data used for construction of the breakage matrix Y according to the procedure given in the theoretical part. Ash contents of the wheat kernel size fractions are presented in the form of the matrix P.

$$B = \begin{bmatrix} 0.3136 & 0.3416 & 0.3698 \\ 0.2835 & 0.2897 & 0.2935 \\ 0.1146 & 0.1015 & 0.0859 \\ 0.1310 & 0.1199 & 0.1077 \\ 0.0406 & 0.0356 & 0.0358 \\ 0.0395 & 0.0377 & 0.0364 \\ 0.0394 & 0.0369 & 0.0363 \\ 0.0378 & 0.0371 & 0.0346 \end{bmatrix}, \ Y = \begin{bmatrix} 0.4422 & 0.4605 & 0.4691 \\ 0.3735 & 0.3729 & 0.3560 \\ 0.0720 & 0.0612 & 0.0585 \\ 0.0520 & 0.0455 & 0.0473 \\ 0.0153 & 0.0137 & 0.0175 \\ 0.0157 & 0.0166 & 0.0185 \\ 0.0130 & 0.0137 & 0.0162 \\ 0.0163 & 0.0159 & 0.0169 \end{bmatrix}, \ P = \begin{bmatrix} 1.4 \\ 1.47 \\ 1.58 \end{bmatrix} \tag{29}$$

The following examples test the possibility of simultaneous control of both input and output PSDs (Example 1) and the input PSD together with the compositional distribution in the output (Example 2).

Example 1.

Following eq. (4) and the matrix B given in the system (29), $o_1 \in (0.3136, 0.3698)$. By choosing, for example $o_1 = 0.33$, by eq. (17) the linear relationship between input fractions is obtained:

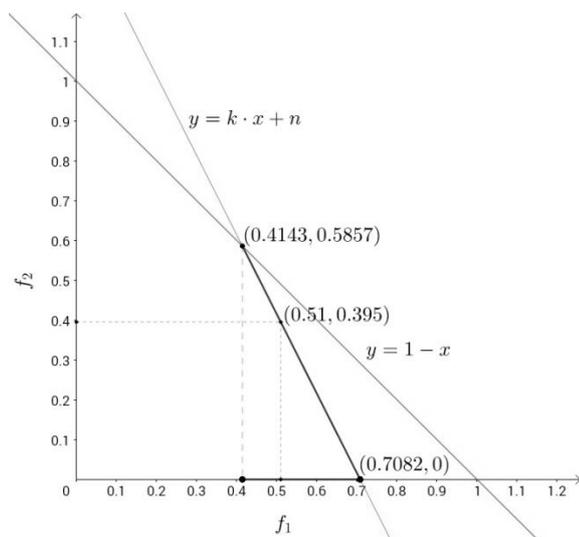$$f_2 = kf_1 + n, \ k = -1.9929, \ n = 1.4113 \tag{30}$$

Figure 1. Graphical representation of the solution (Example 1)

The *x* value of the intersection of this line with $f_2 = 1 - f_1$ line, on one side, and the intersection with *x*–axis, on the other, determine the interval of acceptable values for the first input fraction $f_1 \in (0.4143, 0.7082)$ (Fig. 1). Choosing the value of $f_1 = 0.51$, for example, leads to following input and output PSDs and ash contents in output fractions:

$$f = \begin{bmatrix} 0.5100 \\ 0.3950 \\ 0.0950 \end{bmatrix}, \ o = \begin{bmatrix} 0.3300 \\ 0.2869 \\ 0.1067 \\ 0.1244 \\ 0.0382 \\ 0.0385 \\ 0.0381 \\ 0.0372 \end{bmatrix}, \ p = \begin{bmatrix} 1.9535 \\ 1.8724 \\ 0.8512 \\ 0.5914 \\ 0.6442 \\ 0.7160 \\ 0.5707 \\ 0.6607 \end{bmatrix}. \tag{31}$$

Example 2.

Instead of choosing the weight fraction of some output (Example 1) there is also a possibility to choose ash content in one of the output fractions together with choosing the weight fraction of one of the inputs. Considering the ash content in input fractions *P* (29) and by following the condition (7), the acceptable possible range for the ash content in the first output fraction is:

$$1.9741 \le p_1 \le 2.0043 \tag{32}$$

Let $p_1 = 1.98$. Then, following the procedure described in the theoretical part (8), the possible interval for f_1 weight fraction is determined by the intersections of lines $f_2 = 1 - f_1$ and

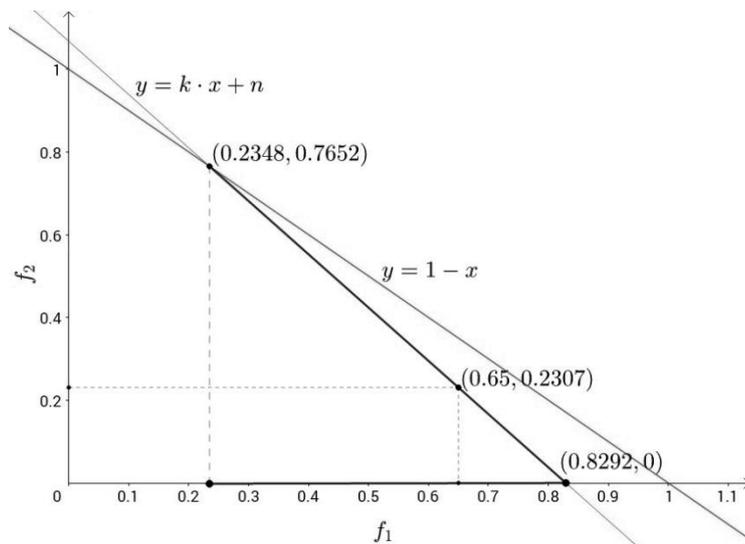$$f_2 = kf_1 + n, \quad k = -1.2873, \quad n = 1.0674 \tag{33}$$

Figure 2. Graphical representation of the solution (Example 2)

So, acceptable solutions are obtained with f_1 in the range 0.2348 to 0.8292 (Fig. 2). For example, if we choose $f_1 = 0.65$, then the input and output PSDs and ash contents in output fractions are determined as:

$$f = \begin{bmatrix} 0.6500 \\ 0.2307 \\ 0.1193 \end{bmatrix}, \ o = \begin{bmatrix} 0.3268 \\ 0.2861 \\ 0.1082 \\ 0.1257 \\ 0.0389 \\ 0.0387 \\ 0.0385 \\ 0.0373 \end{bmatrix}; \ p = \begin{bmatrix} 1.9800 \\ 1.8644 \\ 0.8997 \\ 0.5703 \\ 0.5624 \\ 0.6045 \\ 0.5079 \\ 0.6284 \end{bmatrix} \tag{34}$$

Example 3.

The example 3 aims to determine the input PSD, which would lead to desired ash contents in output fractions. By using appropriate coefficients given in Appendix, values of *B, Y* and *P* from matrices (29), in the expression (11) the following nonlinear relations for f_1, f_2 and f_3 are obtained:

$$f_1(p_1,p_2) = \frac{-0.00687146p_1p_2 + 0.0106p_1 + 0.016p_2 - 0.0255}{-0.00006844p_1p_2 + 0.00031154p_1 + 0.00017846p_2 - 0.00079466} ,$$

$$f_2(p_1,p_2) = \frac{0.0128p_1p_2 - 0.017p_1 - 0.0284p_2 + 0.0393}{-0.00006844p_1p_2 + 0.00031154p_1 + 0.00017846p_2 - 0.00079466}$$

$$f_3(p_1,p_2) = \frac{-0.005999368p_1p_2 + 0.00671872p_1 + 0.0125636p_2 - 0.01461256}{-0.00006844p_1p_2 + 0.00031154p_1 + 0.00017846p_2 - 0.00079466}$$

(35)

Hence, the system (12) consists of three nonlinear inequalities given in the form of (35).

Now, if the ash content values are chosen simultaneously in both output fractions, the solution is represented by the interior of a 3-dimensional area, in which it is not easy to specify the acceptable solution. On the other hand, by using the condition (7), it is possible to choose the ash content in one (arbitrary) output fraction, for example $p_1 = 1.98$. Then, by the system (35), a 2-dimensional illustration for acceptable values of p_2 is obtained (Fig. 3).
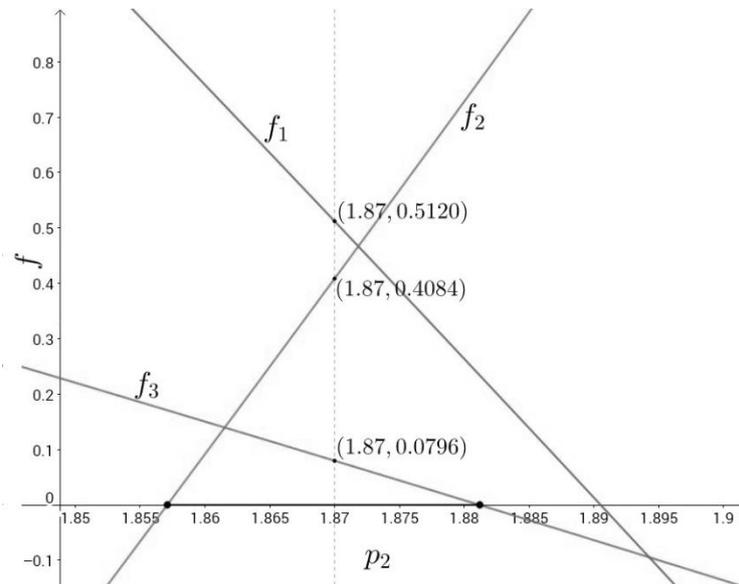

Figure 3. Graphical representation of acceptable values for p_2 (Example 3)

Intersection of the obtained lines with the p_2-axis determines the region where f_1, f_2 and f_3 have positive values, which implies the acceptable range for choosing $p_2 \in (1.8571, 1.8812)$. Let $p_2 = 1.87$, then the input and output PSDs and ash contents in output fractions are:

$$f = \begin{bmatrix} 0.5120 \\ 0.4084 \\ 0.0796 \end{bmatrix}, \ o = \begin{bmatrix} 0.3295 \\ 0.2868 \\ 0.1070 \\ 0.1246 \\ 0.0382 \\ 0.0385 \\ 0.0381 \\ 0.0373 \end{bmatrix}, \ p = \begin{bmatrix} 1.9800 \\ 1.8700 \\ 0.8948 \\ 0.5661 \\ 0.5604 \\ 0.6113 \\ 0.5135 \\ 0.6268 \end{bmatrix}$$

(36)

Alternatively, p_1 value could be chosen according to the condition (7), so that a linear relation between f_1 and f_2 is defined by eq. (6). Again, by taking $p_1 = 1.98$ the acceptable values for the first input fraction $f_1 \in (0.2348, 0.8292)$ are obtained as the intersection (Fig. 4) of lines $f_2 = 1 - f_1$ and:

9

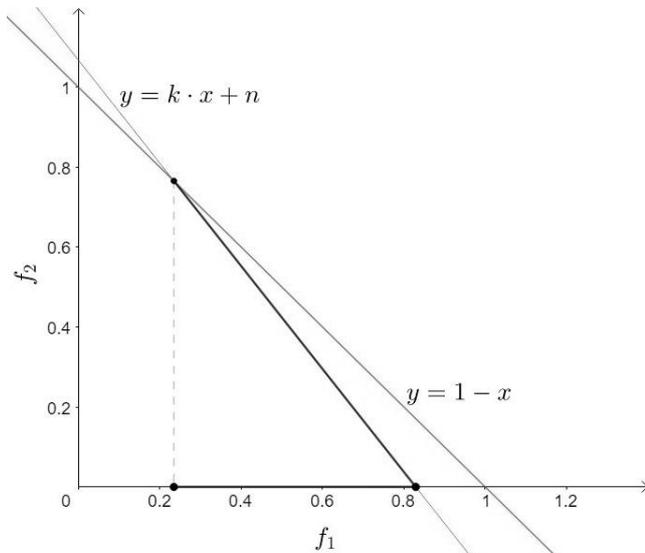$$f_2 = kf_1 + n, \quad k = -1.2873, \quad n = 1.0674 \tag{37}$$

Figure 4. Graphical representation of acceptable values for f_1 (Example 3)

Now, by eq. (11), a monotonic nonlinear function is obtained:

$$p_2(f_1) = \frac{-0.0212 f_1 + 0.5472}{-0.005108 f_1 + 0.2894} \tag{38}$$

and boundaries of the interval for f_1 determine the acceptable range for the ash content in the second output fraction $p_2 \in (p_2(0.8292), p_2(0.2348)) = (1.8571, 1.8812)$. Selection of the same value for $p_2 = 1.87$, as in the first part of this example, of course leads to the same solutions for f, o and p.

If the values for ash contents in all output fractions are set, for example:

$$p = \begin{bmatrix} 1.9678 \\ 1.8621 \\ 0.8302 \\ 0.6329 \\ 0.8346 \\ 0.7294 \\ 0.6357 \\ 0.8608 \end{bmatrix} \tag{39}$$

by using the semilog norm (25) the approximate solution $\bar{p}$ is obtained, with the coefficient of determination r^2 as:

$$f = \begin{bmatrix} 0.4015 \\ 0.2506 \\ 0.3479 \end{bmatrix}, \bar{p} = \begin{bmatrix} 1.9675 \\ 1.8611 \\ 0.8250 \\ 0.6299 \\ 0.8233 \\ 0.7286 \\ 0.6330 \\ 0.8492 \end{bmatrix}, r^2 = \frac{\sum\limits_{i=1}^{m}(p_i - \bar{\bar{p}})^2}{\sum\limits_{i=1}^{m}(p_i - \bar{\bar{p}})^2 + \sum\limits_{i=1}^{m}(p_i - \bar{p_i})^2} = 0.99985 \tag{40}$$

As a consequence of the vector f (the input PSD) the PSD of the output is also determined:

$$o = \begin{bmatrix} 0.3402 \\ 0.2885 \\ 0.1013 \\ 0.1201 \\ 0.0377 \\ 0.0380 \\ 0.0377 \\ 0.0365 \end{bmatrix} \tag{41}$$

Example 4.

The last in this series of examples depicts the case of the simultaneous control of both particle size and the compositional distribution in the output from a milling operation. Different approaches, so called precision and approximation, as well as different pathways within the precision approach have been tested. Considering the precision approach and the fact that arbitrary n-1 outputs could be set, the following procedure will show how the value of ash content in one output fraction together with the contribution of the same output fraction in PSD of the output determine the solution, *i.e.* the particle size and compositional distribution as a consequence of the input PSD. By using values and coefficients given in matrices (29) and Appendix, following relations are obtained:

$$f_1(o_1, p_1) = 168.5665 o_1 p_1 - 384.0149 o_1 + 17.0709$$

$$f_2(o_1, p_1) = -355.9375 o_1 p_1 + 729.8452 o_1 - 20.9073 \tag{42}$$

$$f_3(o_1, p_1) = 167.371 o_1 p_1 - 345.8304 o_1 + 4.8364 .$$

Acceptable values of o_1 and p_1 could be represented in a 3-dimensional space, as a solution of the system of three nonlinear inequalities imposed due to the positivity of input fractions. On the other side, in accordance to eq. (4), the value of the weight fraction of one of the outputs could be set, for example $o_1 = 0.35$ and inserted in relations (42) to get a more useful 2D interpretation shown in Figure 5.
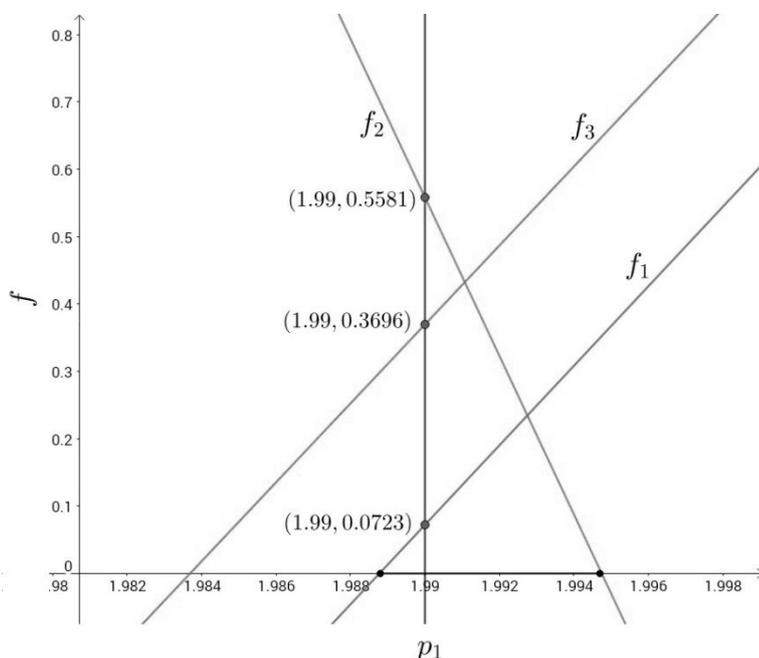

Figure 5. Graphical representation of the solution (Example 4)

Since input fractions must be positive, the acceptable range for $p_1 \in (1.9888, 1.9947)$ is obtained. By choosing $p_1 = 1.99$ the following solution is obtained:

$$
f = \begin{bmatrix} 0.0723 \\ 0.5581 \\ 0.3696 \end{bmatrix}, \; o = \begin{bmatrix} 0.35 \\ 0.2907 \\ 0.0967 \\ 0.1162 \\ 0.0360 \\ 0.0373 \\ 0.0369 \\ 0.0362 \end{bmatrix}, \; p = \begin{bmatrix} 1.99 \\ 1.8979 \\ 0.9489 \\ 0.6043 \\ 0.6385 \\ 0.6964 \\ 0.5973 \\ 0.6864 \end{bmatrix}
\tag{43}
$$

Also, by applying the condition (7) the value of $p_1 \in$ (1.9196, 2.0081) could be set primarily. If $p_1 = 1.99$ then by using the system (42) the acceptable range for the weight fraction of the output $o_1 \in$ (0.3232, 0.3350) (Fig. 6) is obtained.
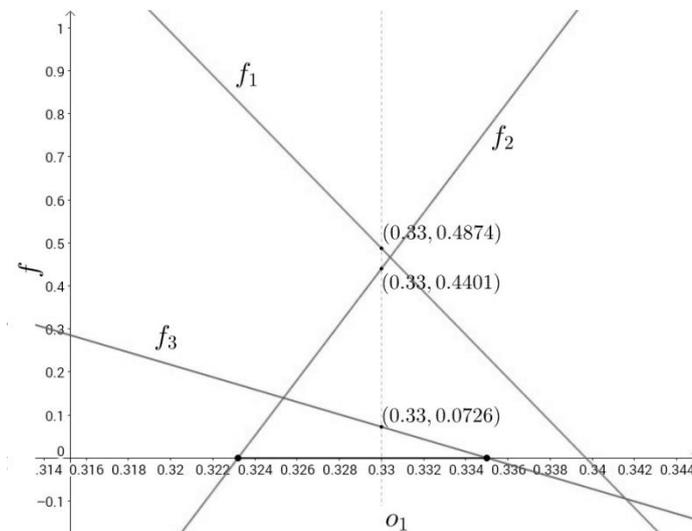

Figure 6. Graphical representation of the acceptable range for o_1 (Example 4)

By choosing, for example $o_1 = 0.33$ the following solution is obtained:

$$
f = \begin{bmatrix} 0.4874 \\ 0.4401 \\ 0.0726 \end{bmatrix}, \; o = \begin{bmatrix} 0.33 \\ 0.2870 \\ 0.1068 \\ 0.1244 \\ 0.0381 \\ 0.0385 \\ 0.0381 \\ 0.0373 \end{bmatrix}, \; p = \begin{bmatrix} 1.9800 \\ 1.8710 \\ 0.8990 \\ 0.5653 \\ 0.56 \\ 0.6125 \\ 0.5145 \\ 0.6815 \end{bmatrix}
\tag{44}
$$

By setting the values for ash contents and weight fractions (PSD) of all output fractions, for example:

$$
o = \begin{bmatrix} 0.3412 \\ 0.2875 \\ 0.1014 \\ 0.1203 \\ 0.0378 \\ 0.0381 \\ 0.0369 \\ 0.0368 \end{bmatrix}, \; p = \begin{bmatrix} 1.9678 \\ 1.8621 \\ 0.8302 \\ 0.6329 \\ 0.8346 \\ 0.7294 \\ 0.6357 \\ 0.8608 \end{bmatrix}
\tag{45}
$$

then by using the semilog norm (27) the input PSD is obtained, which leads to approximate solutions $\tilde{o}$ and $\tilde{p}$, with the coefficient of determination r^2

$$f = \begin{bmatrix} 0.4103 \\ 0.2437 \\ 0.3460 \end{bmatrix}, \bar{o} = \begin{bmatrix} 0.3399 \\ 0.2885 \\ 0.1015 \\ 0.1202 \\ 0.0377 \\ 0.0380 \\ 0.0377 \\ 0.0365 \end{bmatrix}, \bar{p} = \begin{bmatrix} 1.9676 \\ 1.8605 \\ 0.8235 \\ 0.6332 \\ 0.8343 \\ 0.7219 \\ 0.6321 \\ 0.8617 \end{bmatrix}, r^2 = \frac{\sum\limits_{i=1}^{m}(o_i - \bar{\bar{o}})^2 + \sum\limits_{i=1}^{m}(p_i - \bar{\bar{p}})^2}{\sum\limits_{i=1}^{m}(o_i - \bar{o})^2 + \sum\limits_{i=1}^{m}(o_i - \bar{o})^2 + \sum\limits_{i=1}^{m}(p_i - \bar{\bar{p}})^2 + \sum\limits_{i=1}^{m}(p_i - \bar{p_i})^2} = 0.99994 \quad (46)$$

5. CONCLUSION

The reverse breakage matrix approach has shown to be an adequate method for simultaneous control of particle size distributions of the input and output materials, together with the compositional distribution in output fractions during the comminution process. Depending on the demands, two different approaches could be applied. When there is a need to specify the exact values for any of the combination of the input and output size fractions and compositional distribution in output fractions, the precision approach is used. Limitation of this approach is that only certain number of parameters can be chosen freely, and this number is directly determined by dimensions of the breakage matrix. By using the second, approximation approach, the user can demand all of the output size fractions or output compositional distribution. Moreover, a combination of all output fractions and the compositional distribution can be required. However, values for PSDs and compositional distribution that are demanded, must be within an interval that dictates the breakage matrix. As a result, the best-fit particle size distribution of the inlet material is obtained. The proposed model was tested and successfully applied on the examples related to the wheat flour milling. In specific, the model is applicable to non-retention mills with short residence times or wherein a single breakage event takes place. The overall results indicate that that the model could be a useful tool in all grinding process where, besides the size reduction, control of the composition of the output fractions is required.

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SAŽETAK

Kontrola raspodele veličine čestica i hemijskog sastava frakcija mliva tokom mlevnog postupka primenom povratnih matrica usitnjavanja

Nemanja Bojanić[1], Aleksandar Fišteš[1], Tatjana Došenović[1], Aleksandar Takači[1], Mirjana Brdar[1], Kiyoshi Yoneda[2] i Dušan Rakić[1]

[1]Faculty of Technology, University of Novi Sad, Boulevard cara Lazara 1, 21000 Novi Sad, Serbia
[2]Faculty of Economics, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka 814-0180, Japan

(Stručni rad)

Predložen je metod baziran na primeni povratnih matrica usitnjavanja za kontrolu efekata postupka mlevenja na veličinu čestica i hemijski sastav dobijenog mliva. Na primerima vezanim za postupak usitnjavanja pšenice razmatrane su mogućnosti primene i ograničenja predloženog metoda. Pokazano je da se primenom matrica usitnjavanja može predvideti raspodela veličina čestica materijala na ulazu, čijim će se usitnjavanje dobiti željena, unapred zadata raspodela veličina čestica na izlazu, kao i željeni hemijski sastav frakcija izlaza. Pored toga, pokazano je da je moguće istovremeno kontrolisati kako veličine čestica frakcija ulaza i izlaza, tako i hemijski sastav dobijenog izlaznog materijala.

Ključne reči: povratni problem; raspodela hemijskih komponenata; raspodela veličine čestica; mlevenje pšeničnog brašna