Analysis of viscoelastic behavior of a filled elastomer under action of different loads

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Abstract

Mechanical properties of viscoelastic filled polymers strongly depend on temperature and strain rate and vary for several orders of magnitude. During service life, a viscoelastic body, especially carboxy-terminated polybutadiene (CTPB) composite solid rocket propellant grain, is subjected to many stress-inducing loads. Its structural integrity analysis (hereafter: "structural analysis"), unlike elastic bodies, is quite complex and sometimes impossible under the action of just a single load. An even greater problem occurs when multiple different types of loads act simultaneously. This study is based on a complete uniaxial mechanical characterization of a viscoelastic CTPB composite rocket propellant, made in MTI-Belgrade, whose results were used for the analysis of the propellant grain reliability. Through an example, this paper shows a behavior of the viscoelastic propellant grain when it is subjected to extremely different environmental loads at the same time. Similar explicit examples are difficult to found in the literature, except in the form of recommended principles for analysis. It is shown that the tensile strength under the action of fast load due to the pressure may be almost 20 times greater than the tensile strength under the slow temperature load. A probabilistic approach is presented in evaluation the reliability and service life. An example is shown for a rocket propellant grain as a viscoelastic body. The presented principles of the analysis can be applied to any arbitrary viscoelastic body in other areas.

Keywords: composite propellant, viscoelasticity, time–temperature shift factor, ultimate strength, damage, probability of failure.

Mechanical characterization of structural materials is the process of determining their mechanical properties, modulus of elasticity (E), ultimate strength (σ_m), and allowable strain (ε_m) in order to estimate safety factor and resistance of the body to the effects of external loads.

For elastic and isotropic materials, it is assumed that the intensities of all mechanical properties are approximately constant, in the field of small deformations [1], in all ambient conditions or in the whole temperature range of use, as well as in all directions. Relationship between stress and strain is proportional and linear. The safety factor for an elastic body is equal to the ratio between ultimate strength and resultant stress (σ_0):

$$\nu_\sigma(t) = \frac{\sigma_m}{\sigma_0} \qquad (1)$$

For viscoelastic materials, such as elastomers, situation is different. Mechanical properties depend not only on intensity of the load and strain, but also on the type of load and strain rate [2,3]. The problem is even greater if the external loads, that act onto the viscoelastic body, are time dependent, which causes the additional variations of material mechanical properties. Above all, the mechanical properties are highly dependent on temperature. A filled polymer, as a mixture of a pure polymer and solid phase in the form of fine powder, also shows strong viscoelastic properties, as long as the particles of the solid phase are completely covered by the polymer matrix [1–3].

The discussion in this paper refers to the rocket motor propellant grain (pos.2 in Fig. 1) made of viscoelastic composite solid rocket propellant.

Under the continuous stresses, the propellant grain as a viscoelastic body is subjected to the weakening of the mechanical properties, due to the natural aging process and cumulative damage in fatigue [6]. A similar scenario could happen to any viscoelastic body, regardless whether it is the rocket motor propellant grain or any element of a structure from the wide use.

The dependence of the propellant mechanical properties on temperature and strain rate is so strong that

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they may vary by several orders of magnitude, especially if one compares the situations at extremely different temperatures, under the action of totally different loads.

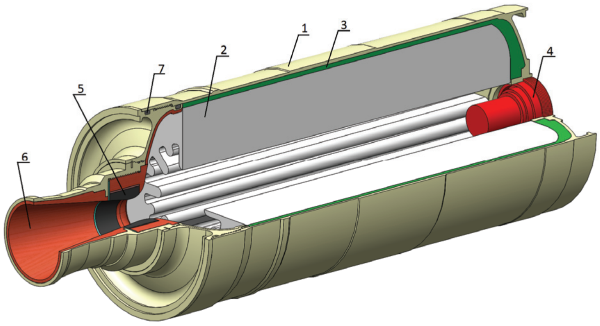

Figure 1. Rocket motor with cast composite propellant grain.

In the mechanical behavior of viscoelastic materials there is an association between the effects of temperature and strain rate [2,3,7,8], which is fortunate, although essentially unexplained [2]. Ultimate strength of the rocket propellant is reduced if the temperature rises and the strain rate is low. In contrast, when the temperature decreases and the strain rate is high, the ultimate strength increases significantly.

What are the consequences of the fact that viscoelastic properties of the composite propellant change due to temperature and strain rate?

Different types of loads produce different strains and strain rates. Mechanical properties of the propellant will vary with different loads. When the ambient temperature is high, the temperature loads that act onto the propellant grain are very low and also very slow, producing low strain rates. Then, the propellant tensile strength is low, in comparison with the case when another fast load acts. In contrast, during the ignition of the rocket motor, especially at low ambient temperature, the pressure in the motor is high, and the operating loads are fast. They produce high strain rate and the propellant tensile strength is entirely different and very high.

A bigger problem occurs in structural analysis of viscoelastic propellant grain when two or more different loads act simultaneously. From the moment it is cast, the propellant grain is continuously under the temperature load that produces quite low strain rate. If, for example, a strong load starts acting due to acceleration of the aircraft carrying the rocket missiles under its wings, the strain rate of the propellant grain would be high. In this situation there is a simultaneous action of two different loads that create different strain-rates and thus also the different mechanical properties of the same propellant. Then it is not possible to estimate the real equivalent values of the propellant mechanical properties, especially ultimate strength as a limit for failure analysis.

In the structural analysis of a viscoelastic propellant grain, a different principle is applied [9–11] compared with the analysis of the elastic body. Instead comparison the constant value of ultimate strength to the resultant stress of all external loads, in order to get the total value of the safety factor, each individual load is considered separately. It is theoretically possible if the linear viscoelasticity is assumed and convolution concept is applied [9]. Then, the Miners principle can be used [6] in accumulation of damage due to sequential or superposed loadings. According to this principle, each external load (L_i) produces some current damage (d_i), equal to the ratio between induced stress (σ_i) and corresponding ultimate strength (σ_{mi}):

$$d_i = \frac{\sigma_i}{\sigma_{mi}} \qquad (2)$$

Sometimes, instead comparing the stress to the ultimate strength, it is better to apply the criteria of the strain comparison to the allowable strain.

In the theory of elasticity the safety factor is defined as the ratio between ultimate strength and resultant stress. Here, for linear-viscoelastic body, the principle is inverse: each individual stress, produced by a singular load has to be compared with the ultimate strength which corresponds exactly to the particular load and its strain rate. It is expected for this ratio to be less than unit. This value represents the currently occupied part of the propellant grain capacity to withstand the failure.

Total current damage can be obtained as the sum of all individual current damages due to the different single loads that simultaneously act on the grain. It is also expected for this summation to be less than unit. Analogically to the principle in theory of elasticity, the safety factor is determined as the reciprocal of the total current damage.

The principle is basically simple, but the analysis may complicate due to the random factors and also due to the time dependence of the external loads. An example of such analysis is presented.

EXPERIMENTAL

Propellant composition and grain production

Composite solid rocket propellant is a viscoelastic material based on polymeric matrix, which is a mixture of about 75–85% solid oxidizer powder embedded in the remaining 15–25% of the matrix [1–4].

A composite propellant composition based on carboxyl-terminated polybutadiene (CTPB) has been produced and tested in the laboratories of Military Technical Institute [4,5], in order to determine its characteristic mechanical properties [12]. The propellant was composed by 14 wt.% of polymer binder (10% CTPB, 0.5% tris(2-methyl-1-aziridinyl)phosphine oxide (MAPO) as a curing agent, 3% circo light oil (CLO) as plasticizer, 0.5% other ingredients) and 86 wt.% solid particles (81% crystalline oxidizer ammonium-perchlorate-AP and 5% Al metal powder).

This propellant composition was used to produce two very similar propellant grains with star shaped central channels (Figures 1 and 2) for different versions of the same rocket missile.

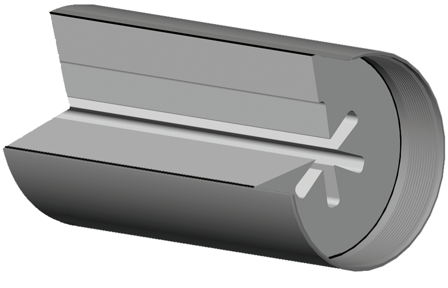

Figure 2. Case bonded propellant grain.

Rocket motor composite propellant grains have been produced by casting the liquid gel mixture at an elevated temperature of about 70 °C into a thin cylindrical rocket motor case (pos.1 in Fig. 1) to get the form of the star shaped propellant grain (pos.2), strongly connected to the case by liner (pos.3). Then, the grain was cured for 120 h in a strictly defined conditions, in the low-humidity chamber at a temperature of 65 °C. Finally, it was cooled down, up to the ambient temperature.

The occurrence of stresses in the propellant grain

Difference between coefficients of thermal expansion of the viscoelastic composite propellant and elastic metal case of a rocket motor is high. Propellant grain (Fig. 2) expands approximately 10 times more intensive than metal case. This leads to high thermal stresses in the propellant grain, which are further increased due to its complex geometric shape and stress concentration. Different variable loads like temperature loads, vibrations, stresses in the grain due to extended polymerisation after curing, acceleration loads, handling, transportation loads, shocks, aerodynamic heating and operating pressure in the rocket motor, may operate at different rates and intensities. Stress state is continuously present throughout the entire rocket motor service life, up to the moment when the rocket is used to perform its mission and the propellant grain is burned out.

Environmental temperature variations affect the changes of thermal stresses in the grain. Those changes can be very large, even on a daily basis. In addition, these cyclic temperature changes can be repeated continuously for a long time, several years or even decades, before the rocket motor is used or removed from use because its reliability has expired.

Test samples preparation for uniaxial tensile tests

In order to check reliability of the rocket motor due to the high stresses caused by the ambient temperature and ignition pressure, and also due to the complex shape of the channel as a strong source of stress concentration, a complete mechanical characterization of the propellant was made and structural analysis of the propellant grain was conducted, including failure analysis.

During the grain manufacturing, propellant blocks for tensile test specimens production were also cast and placed into a dry environment at 20 °C. Afterwards the specimens were cut from the blocks before testing.

Test procedure and results of measurements

Uniaxial constant strain-rate tests were made using the tensile tester Instron-1122 and well known Jannaf-C specimens [1,8,9,13]. Tensile tests were conducted, in accordance with well established test procedure [4,10], in ten degrees steps in the range between –50 and 50 °C, with constant cross-head speeds in the range between 0.2 and 1000 mm/min. In addition, after the complete mechanical characterization of the fresh propellant, some periodic control tests were made later at standard conditions (cross-head speed R = 50 mm/min, temperature t = 20 °C).

The purpose of the complete mechanical characterization was to determine characteristic master curves for the mechanical properties: initial modulus (E), ultimate tensile strength (σ_m) and the strain at maximum stress (ε_m). The all three features (σ_m, ε_m, E), in logarithmic scales, normalized by factor (T_0/T), were plotted *versus* reduced time (ξ), which is the reciprocal of the strain rate, corrected by the time–temperature shift factor: $\log\xi = \log(1/Ra_T)$.

The same well known test procedure of mechanical characterization was applied as it was described in detail in our previous papers [4,10]. Therefore, a detailed description of the measurements is not presented here, nor the procedure of data analysis. Only the necessary test results are presented to explain the thesis in this paper.

RESULTS AND DISCUSSION

Mechanical properties presentation

Mechanical properties of CTPB composite rocket propellant, as well as the properties of typical viscoelastic materials, are strongly dependent on temperature and strain rate. Strain (ε) is a dimensionless feature, as a ratio of two length sizes, elongation (Δl) and basic length (l_0) of a body. Hence, the strain rate dimension ($R = \mathrm{d}\varepsilon/\mathrm{d}t$) is equal to the reciprocal of time.

Therefore, the strain rate is basically time effect, so the propellant mechanical properties depend on temperature and time.

Fortunately, there is a correlation between time and temperature [1–4] which allows to reduce the dependence on two different features into a dependence on only one variable. For example, if the strain rate (time influence) is changed for a certain value (ΔR) at arbitrary temperature (t = const.), the effect should be the same to a certain temperature change (ΔT) at an arbitrary constant strain rate (R = const.).

The most commonly used term for the correlation between the influences of time and temperature is Williams–Landel–Ferry (WLF) equation [3,9,14]:

$$\log a_T = -\frac{C_1(T-T_0)}{C_2+T-T_0} \tag{3}$$

Time–temperature shift factor (a_T) is one of the most important values in the structural analysis of viscoelastic materials because it allows the temperature impact to be converted to the time influence, which makes the analysis much easier. Physical meaning of this factor can be seen in Fig. 3, where the ultimate strength dependences on the strain rate are shown for all test temperatures.

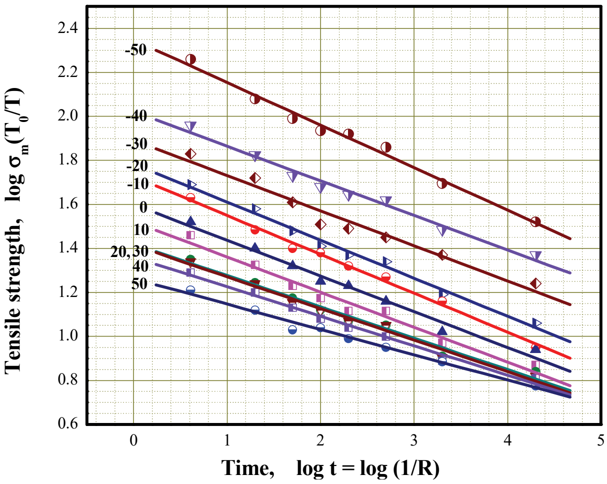

Figure 3. Tensile strength vs. temperature and strain rate.

Simply speaking, at each individual temperature in the range between –50 to 50 °C, at 10 °C increments, the CTPB propellant specimens were tested in eight different constant rate modes. Functional dependencies of the ultimate strength *vs.* strain rate have been obtained. Linear regressions were obtained using commercial program Origin for the analysis of diagrams. These dependencies are approximately linear at all temperatures in the range. Moreover, if we suppose that the impact of nonlinear viscoelasticity of the propellant could be neglected, it seems that all these linear dependencies could be considered as approximately mutually equidistant.

When one of these test temperatures is chosen as a reference one, usually the standard ambient temperature (T_0 = 20 °C), horizontal distance between an arbitrary temperature (T) and the reference temperature (T_0) is equal to the value of $\log a_T(T)$. Factor $a_T(T)$ is different for each individual temperature and its value corresponds to the expression (3). For the tested CTPB propellant, the dependence $\log a_T$ *vs.* T is shown in Fig. 4.

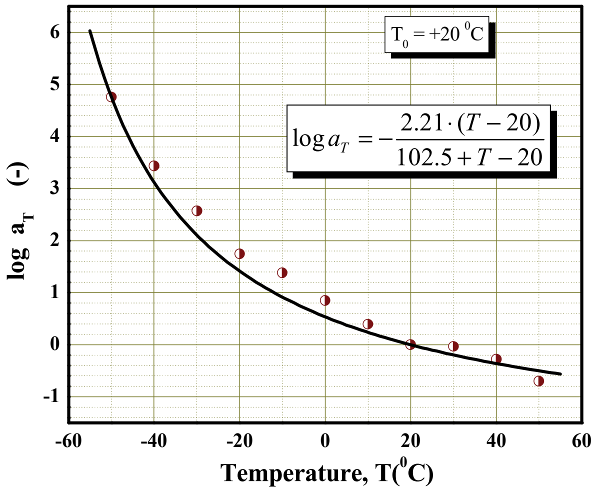

Figure 4. Temperature dependence of the time–temperature shift factor.

When different regression lines (Fig. 3) are moved horizontally, along the time axis to meet the reference line, they overlap creating a single curve termed "master curve" [3,10–12], Fig. 5. It is customary (in all cited literature) to prepare the scale of the abscissa to represent both influences, temperature and time. Ins-

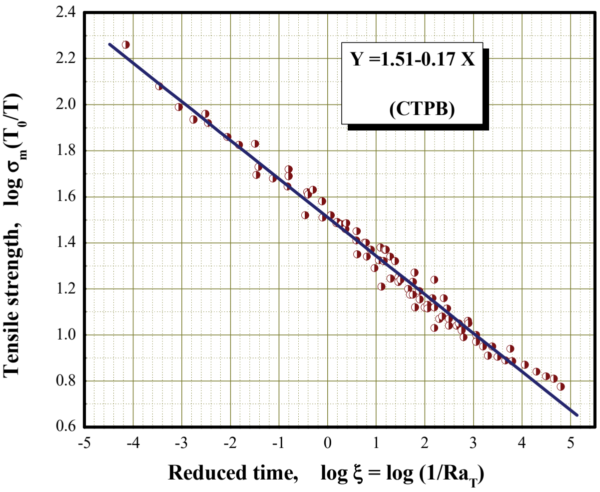

Figure 5.Ultimate strength master curve for CTPB composite propellant.

tead of reciprocal of the strain rate ($t = 1/R$), which has the dimension of real time, a single variable “reduced time”($\xi = 1/(Ra_T) = t/a_T$) is introduced, which is a combination of temperature and time (strain rate) [4,10].

Relationship between time and temperature is evident because moving along the timeline corresponds to the temperature change.

Finally, the tensile strength of the composite propellant as a viscoelastic material is displayed in the form of master curve which includes the strength dependence in all load conditions. This amounts to a total of 88 different points, each of them corresponding to one regime (11 different temperatures between –50 and 50 °C at 10 °C increments, multiplied by 8 different cross-head rates of the tensile tester: 0.2, 2, 5, 10, 20, 50, 100, 200 and 1000 mm/min, Fig. 5). Since the elastic forces in polymers are proportional to absolute temperature [1,3,8], tensile strength is normalized by factor (T_0 / T).

For the analysis in this paper, only the master curve of tensile strength is needed (Fig. 5), and its mathematical representation (4), as well as the value of the time–temperature shift factor (a_T , Fig. 4), expression (5):

$$\log(\sigma_{\mathrm{m}} \frac{T_0}{T}) = 1.51 - 0.17 \cdot \log \xi = 1.51 - 0.17 \log\left(\frac{1}{Ra_T}\right) \quad (4)$$

$$\log a_T = -\frac{2.21(T-20)}{102.5 + T - 20}, \; T\text{: absolute temperature} \quad (5)$$

Ultimate strength dependence on the type of load

Solid propellant grain (Fig. 1), from the time it is cured until the moment when it will be used and burned, suffers continuous stress due to the effects of temperature difference between the casting and ambient temperature. When the grain finally starts to burn, at the same time it suffers the loads due to the pressure in the rocket motor and also acceleration load, etc.

Strain rate after curing, made by the temperature difference between the grain temperature and zero stress temperature, is quite small, because the time period of propellant grain cooling down after curing, up to the ambient temperature, is considerably long. In another type of load, due to the pressure during the rocket motor ignition, the pressure is rapidly growing up and the strain rate is high.

Different strain rates affect the size of the propellant tensile strength.

Difference between the influences of these two values of strain rates can be seen in the case of tangential strain components in the hollow tube grain channel (Fig. 6).

For the sake of comparison, let us consider the equations obtained in the elastic analysis, for tangential strain in the grain channel, due to the influence of temperature and the effects of increasing pressure during the rocket motor ignition. For star pointed channel, the expressions for the hollow tube channel have to be corrected using the stress concentration factor.

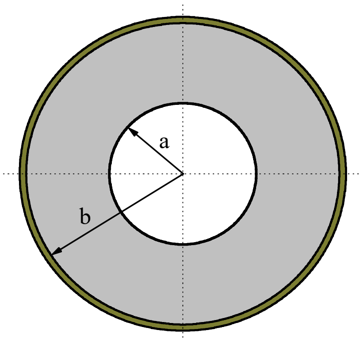

Figure 6. Hollow tube grain.

In the case of pressure loading, the elastic equation for the tangential strain in the grain channel [1,7–9,11,15] has the following form (3):

$$\varepsilon_\theta(p) = -\frac{(1+\nu)p}{E}\left\{1 - 2\nu - \frac{2(1-\nu)KM^2}{M^2-1}\left[1 - \frac{2(1-\nu)}{\Omega}\right]\right\} \quad (6)$$

For the temperature loading, the following expression is taken from the same references:

$$\varepsilon_\theta(T) = \alpha(1+\nu)\Delta T - 2KM^2\Phi(1-\nu)\Delta T \quad (7)$$

Titles and explanations of all the features in these expressions are given in the Table 1. The features Ω and Φ can be obtained using the expressions (8) and (9), taken from the same sorce:

$$\Omega = 1 + (1-2\nu)M^2 + \frac{E}{E_{\mathrm{c}}}\frac{b}{h}\left(M^2 - 1\right)\frac{1-\nu_{\mathrm{c}}^2}{1+\nu} \quad (8)$$

$$\Phi = \frac{\alpha\left(1-\nu\right) - \alpha_{\mathrm{c}}\left(1-\nu_{\mathrm{c}}\right)}{\Omega} \quad (9)$$

For selected propellant grains (Figs. 1 and 2), the geometric and physical properties are shown in the Table 1.

Since the load conditions were not known in advance, the propellant modulus (*E*) in the Table 1, dependent on the strain rate, had to be approximately determined through several iterations.

Tangential strains in the grain channel due to the pressure and temperature loads have been considered in conditions of the lowest extreme temperature of the rocket motor usage (–40 °C). During the rocket motor ignition, the operating pressure of approximately 180 bar is achieved very fast, in about 10 ms, and the strain rate is high. On the other hand, we have a very slow load due to the temperature. The propellant grain was cured at 65 °C and it can be assumed that there were

Table 1. Propellant grain properties

Notation	Unit	Value	Feature
b	mm	148	Outer radius of the grain
a	mm	65	Inner radius of the grain
M	–	2.28	Outer/inner radius ratio
h	mm	2.0	Case thickness
K	–	3.1	Stress concentration in the star perforated grain
T_0	K	293	Reference temperature
ν	–	≈0.5	Propellant Poisson's ratio
ν_c	–	≈0.3	Case Poisson's ratio
E	daN cm^{-2}	≈600	Propellant modulus
E_c	daN cm^{-2}	2.1×10^{6}	Case modulus of elasticity
α	°C^{-1}	0.93×10^{-4}	Propellant coefficient of thermal expansion
α_c	°C^{-1}	0.11×10^{-4}	Case coefficient of thermal expansion
C_1	–	2.21	Time-temperature shift factor coefficients
C_2	–	102.5	

no stresses at this temperature. It is also assumed in this example, very rough, that after curing the grain was cooled down for about 24 h up to the considered temperature (–40 °C), although this cooling down in reality is much more slowly, which further reduces the strain rate due to the temperature load.

Replacing the values from the Table 1 into the expressions (8) and (9) we have Ω = 1.054 and Φ = = 0.368×10^{-4}. From the expressions (6) and (7) we get the values for the strains due to the different effects of pressure and temperature:

$$\varepsilon_\theta(p)\approx 0.0885 \tag{10}$$

$$\varepsilon_\theta(T)\approx 0.0476 \tag{11}$$

Strain rate is defined as the ratio of strain and the time needed to achieve the strain:

$$R=\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\approx\frac{\varepsilon}{t} \tag{12}$$

Strain rates during the rocket motor ignition and during the propellant grain cooling down after curing are shown in expressions (13) and (14):

$$R(p)=\frac{\varepsilon_\theta(p)}{t(p)}=\frac{0.0885}{10\times10^{-3}}=8.85\ \mathrm{s}^{-1} \tag{13}$$

$$R(T)=\frac{\varepsilon_\theta(T)}{t(T)}=\frac{0.0476}{24\ \mathrm{h}}=\frac{0.0476}{24\times60\times60}= =0.551\times10^{-6}\ \mathrm{s}^{-1} \tag{14}$$

$$\begin{aligned}\log R(p)&=\log 8.85=0.947\\ \log R(T)&=\log(0.551\times10^{-6})=-6.259\end{aligned} \tag{15}$$

Time-temperature shift factor for the rocket motor lowest working temperature (–40 °C):

$$\log a_T=-\frac{2.21(T-T_0)}{102.5+T-T_0}=-\frac{2.21(233-293)}{102.5+233-293}= =3.12 \tag{16}$$

Reduced times for the pressure and temperature loads, on the abscissa of the tensile strength master curve (Figure 3) are different:

$$\log\xi=\log(\frac{1}{Ra_T}) \tag{17}$$

$$\log\xi_\theta(p)=\log(\frac{1}{R(p)a_T})=-0.947-3.120=-4.067$$

$$\log\xi_\theta(T)=\log(\frac{1}{R(T)a_T})=6.259-3.120=3.139$$

This difference between the two reduced times under the effects of temperature and pressure is significant. In these two cases, the tensile strengths can be calculated from the master curve equation in the Figure 3:

$$\log(\frac{T_0}{T}\sigma_\mathrm{m})=1.51-0.17\log(\frac{1}{Ra_T}) \tag{18}$$

The ultimate strength of the propellant under the pressure load:

$$\log(\frac{293}{233}\sigma_\mathrm{m}(p))=1.51-0.17(-4.067)=2.201$$

$$\sigma_\mathrm{m}(p)=126.32\ \frac{\mathrm{daN}}{\mathrm{cm}^2} \tag{19}$$

The ultimate strength of the propellant under the temperature load:

$$\log\left(\frac{293}{233}\sigma_{\mathrm{m}}(T)\right)=1.51-0.17(3.139)=0.976$$

$$\sigma_{\mathrm{m}}(T)=7.52\ \frac{\mathrm{daN}}{\mathrm{cm}^2} \qquad (20)$$

The ultimate strength under the pressure load (19) is 17 times bigger then the strength in the case of temperature load (20). In other circumstances, this difference may be even greater.

This result seems rather strange. However, it is possible to explaine it by an example from everyday practice. Lets stretch a piece of bubble gum at high speed. It will break very fast with little elongation (strain) and breakout forces (tensile strength) will be relatively large. In contrast, a small force will also lead to a rupture, if we keep stretching slowly, but the elongation will be high. The basic question that arises is: which breakout force (tensile strength) is representative as a failure criterion when two or more different forces act simultaneously onto a body?

Reliability and failure criteria

Simultaneous effects of multiple loads on a viscoelastic body

On the basis of the previous example, it is not possible to determine the safety factor for a viscoelastic body in the same way as in the theory of elasticity. For an elastic material the ultimate strength is treated as a constant value, but for a viscoelastic body, real stresses and strains depend on the type of load, strain rate and environmental conditions.

The safety factor for an elastic body is equal to the ratio of ultimate strength (σ_{m}) to the stress (σ_0) as a total result of a number of loads.

On the contrary, when a number of different time dependent loads $L_1(t),L_2(t),L_3(t),...$ act onto a viscoelastic body at the same time, they produce the same number of correspondent different strain rates, $\dot{\varepsilon}_1(t),\dot{\varepsilon}_2(t),\dot{\varepsilon}_3(t),...$ Then, the ultimate strengths that correspond to these loads are different: $\sigma_{\mathrm{m1}}(t),\sigma_{\mathrm{m2}}(t),\sigma_{\mathrm{m3}}(t),...$

There is no satisfactory mathematical or physical model that could include simultaneous impact of all external loads together. For that reason, a similar principle is accepted here as for the cumulative damage analysis [1,15–18], which was first applied in Miners consideration of material fatigue [6]. According to this principle the effects of individual loads have to be analyzed separately, using the concept of convolution [9], which is valid for linear-viscoelastic materials. Each of the individual time dependent loads creates a certain current damage, equal to the ratio between real stress and its appropriate ultimate stress. It is also a time-variable feature and it does not represent a final permanent damage, but only a current weakening of the total body resistance against the failure.

Step by step, the first current damage is equal to the ratio between the first stress $\sigma_1(t)$ and the corresponding first tensile strength $\sigma_{\mathrm{m1}}(t)$. Then the same principle applies to the second load, etc. Theoretically, there will be no failure as long as the total sum of the individual current damages is less than unit.

$$d(t)=d_1(t)+d_2(t)+d_3(t)+... \qquad (21)$$

$$d(t)=\frac{\sigma_1(t)}{\sigma_{\mathrm{m1}}(t)}+\frac{\sigma_2(t)}{\sigma_{\mathrm{m2}}(t)}+\frac{\sigma_3(t)}{\sigma_{\mathrm{m3}}(t)}+...\le 1 \qquad (22)$$

Total current damage $d(t)$ is time dependent and finally equal to the sum of the all individual current damages. The safety factor of the body is the reciprocal of the total current damage:

$$\nu(t)=\frac{1}{d(t)} \qquad (23)$$

The case with two totally different loads in rocket motor, pressure and temperature, shows that for the structural analysis of a propellant grain, as well as for an arbitrary viscoelastic body, it is necessary to know all its mechanical properties and their dependences on temperature and strain rate.

In addition to the tensile strength, allowable strain, tangent modulus, especially “relaxation modulus” [1–4,8–12], for a viscoelastic material it is necessary to know the natural aging properties [4,16–20], as well as the impact of cumulative damage [10,18,21].

Time variability of safety factor

There are difficulties in terminology how to explain what part of total resistance of a body is engaged under the action of external loads. This problem is especially strong in viscoelastic materials. In addition to being dependent on external conditions, the stresses may remain in the viscoelastic body even after the load termination, although they can also relax over time. Relative current value that indicates occupied part of the maximum resistance of a viscoelastic body is named “current damage” [10]. This value does not reflect permanent injury of the body.

In the rocket motor, “current damage” of the propellant grain is a time-variable because it depends on the environmental loads that vary over time.

Some loads can quit and the stress field in the body will disappear. A part of their influence may remain for a while, but their influence will be meaningless, because the stresses in the viscoelastic body usually relax.

The following example was discussed in reference [10]: a combat aircraft with rockets mounted under its wings, returned to its base without launching. The rocket motor had been under the stresses due to different loads: vibrations of the wings, aerodynamic heating, axial acceleration of the aircraft, and also the most intensive load due to the temperature difference. Each individual loads cause their partial current damages. In the moment when the aircraft is in the air, the total current damage is the sum of four partial damages:

$$d(t)=d_1(t)+d_2(t)+d_3(t)+d_4(t) \tag{24}$$

When the aircraft lands back, the most of the stresses due to the first three loads dissapear and the residual stresses relax after a while and return to zero. Then, the total current damage depends only on the temperature difference:

$$d(t)\approx d_4(t) \tag{25}$$

The example shows the time dependence of the "current damage", in accordance with the action of time-dependent loads. Theoretically, if the total current damage was always less then unit over the time, $d(t)\le 1$, there would not be any propellant grain failure. After the plane lending in the previous example, the first three stresses will disappear and the only remaining stress will be caused by temperature difference, and the correspondent current damage of the propellant grain will be equal to $d_4(t)$. The cumulative damage due to the disappeared loads, that were acting shortly, is almost negligible. Finally, the remaining current damage itself is too small to exceed unit, until the appearance of a new larger load or until the deterioration of the viscoelastic propellant grain due to natural aging.

Unlike the concept of "current damage", in the analysis of viscoelastic materials there is a very important feature named "cumulative damage". It represents a permanent injury of the viscoelastic body due to the previous effects of loads over the service life, especially cyclic loads of high frequency [18,22].

The time dependence of the current damage is clearly recognized when the external load, acting on the propellant grain, is a time variable. This is evident in the case of temperature stresses in the grain, because the ambient temperature continuously oscillates over time, passing a complete sinusoidal cycle every day, and also a seasonal cycle each year.

An example of current damage time distribution is shown in Fig. 7, during the first two years after the propellant grain production, whose properties are defined in Table 1. The reason for global increasing of current damage is weakening of the polymer due to the natural aging and cumulative damage.

High jumps of current damage in regular intervals are the result of temperature jumps. The physical model of temperature change was made on the basis of statistical data, but an assumption has been introduced that temperature shocks occur once a week, jumps or falls, respectively.

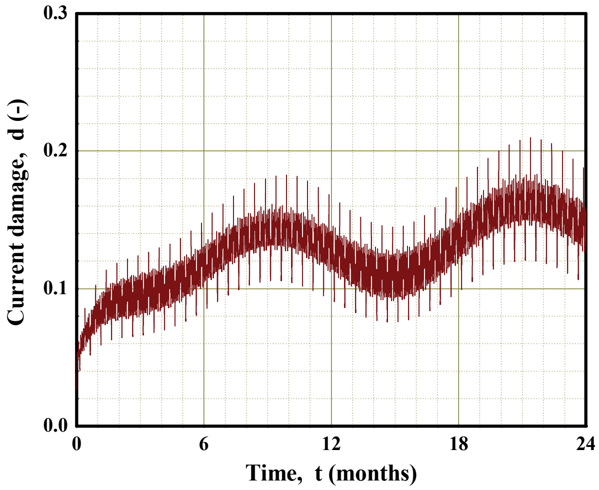

Figure 7. Two-year current damage distribution of the propellant grain due to environmental temperature.

Finally, the safety factor of the propellant grain as a viscoelastic body has to be considered as a time-dependent feature at least for two reasons: load variations and falling off the allowable mechanical properties of the propellant.

Probabilistic approach to estimate the reliability

Large variations of the current damage due to the temperature load in Fig. 7 are not critical because they are much less than unity. However, when two or more variable loads act at the same time, for example temperature, pressure and acceleration load, total value of current damage vary even more. Then, it is almost impossible to define the real total intensity of the current damage.

In the case of temperature load alone, a group of important authors recommended probabilistic methodology [17,18,23] for evaluation reliability instead of safety factor of the propellant grain. Similarly, it is possible to apply the probability approach in the case when a number of different loads operate simultaneously.

A good example for structural analysis of a viscoelastic body may be typical tangential stress (σ_θ) in the circular channel of axisymmetrical cylindrical propellant grain.

Let us assume that the probability of a grain failure due to the first single load $(\sigma_{\theta 1})$ is equal to the probability that the ultimate strength that corresponds to the first load is less than calculated value of tangential stress:

$$P_{f1}=P\left(\sigma_{m1}\le\sigma_{\theta 1}\right) \tag{26}$$

Similarly, in the case of the second load ($\sigma_{\theta 2}$) we have:

$$P_{f2} = P(\sigma_{m2} \le \sigma_{\theta 2}), \text{ etc.} \tag{27}$$

Finally, it is necessary to define a model for total probability of failure. The simplest assumption is that the events 1, 2,…,*n* are independent, and probability of grain failure is the sum of individual probabilities:

$$P_f = P_{f1} + P_{f2} + \ldots + P_{fn} \tag{28}$$

If it is assumed that the probabilities of failure due to individual loads are dependent, the model will be slightly more complicated, but still quite simple. This is an area which deals with structural analysis.

This approach requires a statistical analysis of all variables, at first mechanical properties of the propellant (or arbitrary viscoelastic material), and then for all the real loads and stresses.

Propellant grain reliability

During the service life of a viscoelastic body, current damages appear and disappear along with the time-dependent loads. If the stresses due to concentrated loads do not exceed the allowable limit of the viscoelastic material, they will stop along with the loads or they will quickly relax. However, it remains for ever a permanent impact of the cyclic loads named cumulative damage [10,18,21,22].

The impact of cyclic loads is usually small but it increases with the number of repeated load cycles, thus increasing the probability of crack or unbonding, or dewetting the particles of oxidizer powder embedded into the polymeric matrix.

Let consider the reliability of the propellant grain in the case of temperature cycling. Every day is a sine cycle. At the end of the cycle, the probability of failure (P_f) may be very small, but it cannot be neglected. The probability of opposite event is ($\bar{P}_f$) and it is equal to the probability that the grain will survive:

$$\bar{P}_{fi} = 1 - (P_f)_i \tag{29}$$

After *n* cycles, the reliability is equal to the product of *n* of single probabilities of surviving. It is necessary to determine each individual probability ($\bar{P}_{fi}$) at the end of all previous temperature cycles:

$$R_n = \bar{P}_{f1}\bar{P}_{f2} \cdots \bar{P}_{fn} \tag{30}$$

$$R_n = (1 - P_{f1})(1 - P_{f2}) \cdots (1 - P_{fn})$$

$$R_n = \prod_{i=1}^{i=n} (1 - P_{fi}) \tag{32}$$

In the case of the propellant grain defined in Table 1, the results of structural analysis are shown in Fig. 8. Probability of failure and reliability are shown in parallel. Some periodic irregularities in the probability curve are result of mathematical representation of temperature and simulated periodic shocks.

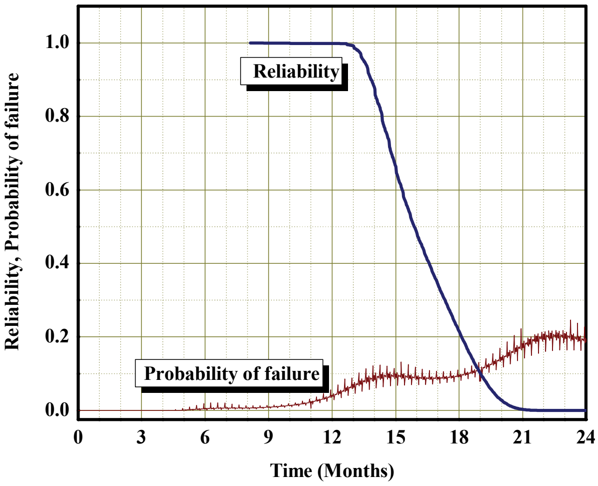

Figure 8. Time distribution of reliability and probability of failure.

The advantage of probabilistic approach to the analysis is seen on a smooth and clear reliability curve. When the reliability fall low enough, below the acceptable limit, it will be the moment which determine the end of the service life of the viscoelastic body.

Although it seems that the probability of failure is a good representative of the propellant grain safety, it is not so. Reliability is much better indicator for making a decision whether the grain is enough reliable or the service life is going to expire. In Fig. 8 the grain reliability falls below the allowable limit (for instance, $R \le 0.95$), much earlier ($t_f \approx 13.5$ months) than the probability of failure reaches high values (Probability of failure $P \ge 0.1$ after 18 months).

CONCLUSION

This paper discusses the complexity of determination the safety factor of a viscoelastic body due to the high dependence of its mechanical properties on external conditions, especially in the cases where different loads act simultaneously onto the body.

It is shown that a viscoelastic material may have extremely different mechanical properties depending on the type of load that act onto the viscoelastic body. On the basis of uniaxial mechanical characterization of viscoelastic composite rocket propellant, which have been made over the wide range of different modes of uniaxial tensile tests, the master curves were obtained that represent the mechanical properties dependence on temperature and strain rate. After that, using a mathematical model for the calculation of strain rate under the action of extremely different loads, due to the ambient temperature and the ignition pressure in the rocket motor, an example is shown where the tensile strength of the material vary up to 20 times.

The principle of adding the effects of simultaneous different loads is presented, resulting in different behaviour of the viscoelastic rocket propellant at the same time, depending on the type of load. This principle of determining the safety factor is different from the principle of analysis the elastic body.

This paper shows how difficult is to assess the reliability of the viscoelastic body subjected to variable loads, when the safety factor varies, while at the same time its variations are increased due to the variations in mechanical properties of the material, caused by changes in the loads. This problem is solved applying the probabilistic approach.

Nomenclature

C_1, C_2	Constants
a_T	Time-temperature shift factor
$d, d(t)$	Current damage, time dependent current damage
E	Modulus, $\mathrm{daN}/\mathrm{cm}^2$
l_0	Basic length, mm
L	Load, daN
p	Pressure, bar
P	Probability
P_f	Probability of failure
R	Strain rate, s^{-1}
R_n	Reliability after n cycles
T, T_0	Temperature, reference temperature, °C
T_g	Glass transition temperature, °C
t	Time, s
ε	Strain, %
ε_m, ε_θ	Ultimate (allowable) strain, tangential strain, %
$\dot{\varepsilon} = \mathrm{d}\varepsilon / \mathrm{d}t$	Strain rate, s^{-1}
ξ	Reduced time, s
ν, ν_σ	Safety factor, Safety factor based on stress criterion
$\sigma, \sigma_\mathrm{m}, \sigma_\theta, \sigma_0$	Stress, strength (ultimate stress), tangential stress, resultant stress, daN/ /cm^2

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ИЗВОД

АНАЛИЗА ВИСКОЕЛАСТИЧНОГ ПОНАШАЊА ПУЊЕНИХ ЕЛАСТОМЕРА ПОД ДЕЈСТВОМ РАЗЛИЧИТИХ ОПТЕРЕЋЕЊА

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(Научни рад)

Структурна анализа вискоеластичних тела је сложенија од анализе еластичних тела, јер механичке особине вискоеластичних материјала значајно зависе од температуре и брзине деформације, и мењају се у распону од неколико редова величина. То је важно ако је вискоеластично тело изложено утицају више различитих оптерећења. Војнотехнички институт (ВТИ) у Београду и Војна академија се баве овом проблематиком неколико деценија, кроз анализу погонских пуњења ракетних мотора. Композитна чврста погонска материја је смеша полимерног горива, прашкастог оксидатора и извесних додатака. Полимер даје овој смеши наглашена вискоеластична својства. Смеша се лије у калуп у стању гела, а након умрежавања добија тражени облик чврстог погонског пуњења, које је надаље непрестано под дејством промењљивих оптерећења. Издржљивост ове структуре је веома важна за поузданост ракете. Анализа поузданости вискоеластичног тела је сложена чак и када делује само једно оптерећење. Када делује више оптерећења истовремено, на свако од њих материјал реагује другачије. Током припаљивања ракетног мотора, при врло брзом оптерећењу услед притиска, чврстоћа горива је висока. Истовремено, на пуњење делује споро температурско оптерећење и тада је чврстоћа горива ниска. Не постоји добар физички модел који би дефинисао еквивалентну чврстоћу горива за оба случаја оптерећења. У овом раду, резултати комплетне једноосне механичке карактеризације композитног вискоеластичног ракетног горива, израђеног у ВТИ-Београд, искоришћени су за анализу поузданости и века употребе једног погонског пуњења ракетног мотора. Кроз пример је приказана реакција вискоеластичног горива на две врсте различитих оптерећења. Затезна чврстоћа горива при брзом оптерећењу услед притиска је скоро 20 пута већа од затезне чврстоће при дејству спорог температурског оптерећења. Зато се дејства појединих оптерећења засебно разматрају и затим сабирају, применом принципа конволуције и претпоставке о линеарној вискоеластичности. Слични примери се тешко могу наћи у литератури. Разматрана је примена теорије вероватноће за процену поузданости и века употребе. Приказани су резултати анализе погонског пуњења ракетног мотора у којој су као улазни подаци коришћени резултати механичке карактеризације горива. Сличан приступ може да се примени код структурне анализе произвољног вискоеластичног тела.

Кључне речи: Гориво композитно • Вискоеластичност • Температурско–временска корелација • Затезна чврстоћа • Оштећење • Вероватноћа лома